

Influence of Scoured Beds on the Characteristics of Seepage  
Underneath Water Structures

تأثير بيارة النحر على خصائص التسرب تحت المنشآت المائية

By

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الخلاصة: - يقدم هذا البحث دراسة لتأثير وجود النحر المحلي المتكون خلف المنشآت المائية على خصائص التسرب أسفل هذه المنشآت وذلك باستخدام نظرية العناصر المحدودة. للقيام بهذه الدراسة تم فرض خمسة حالات عامة مختلفة لعمق بيارة النحر. لاستخدام نظرية العناصر المحدودة تم الاستعانة بالعناصر البسيطة مثل المثلث الثلاثي الارتباط والشكل الرباعي العام الرباعي الارتباط. من خلال النتائج التي تم الوصول إليها تم استخلاص دراسة مقارنتية لخصائص التسرب للحالات الخمسة التي تم تناولها وذلك على شكل منحنيات. من أهم النتائج أن قوة رفع المياه المتسربة للمنشآت تتناسب عكسياً مع ازدياد عمق بيارة النحر بينما يزداد معدل الهباء المتسرب من خلال التربة مما يؤثر على اتزان المنشأ حيث يزداد انحدار الميل الهيدروليكي للسريان خلال التربة وبالتالي تزداد خطورة ظاهرة الفوران التي يميزها انهيار المنشآت المائية.

ABSTRACT - In this paper, the effect of the scoured beds on the characteristics of seepage underneath heading up structures, has been studied numerically using Finite Element technique. Five assumptions of the eroded beds have been considered to achieve the behaviour of flow under the structure. The three and four node isoparametric elements have been used. From the obtained results a complete comparative study has been illustrated. It is observed that the depression of the downstream bed decreases the uplift pressure acting on the floor of the structure while the rate of seepage flow increases. The floor creep velocities and exit gradients for the studied cases have been illustrated.

#### INTRODUCTION

The stilling basin of a weir is designed to dissipate the energy of the water flowing over it. However, in spite of all care taken in designing the stilling basin, there is a possibility of erosion occurring downstream of the structure. The shape and extent of scour depend upon the Froude number of flow over the weir [2,3,4].

The tail erosion changes the boundary of the domain and results in redistribution of pressures under the apron of the weir and the exit gradient in the downstream. Analysis of seepage under hydraulic structures laid on porous medium, which has been subsequently subjected to scour, has not previously been done.

The problem of flow under a flat bottom weir with a vertical sheet pile at the downstream end, founded on a homogeneous and isotropic soil of infinite depth, has been treated numerically using Finite Element technique. Five different shapes commencing from the toe of the weir have been considered. The case of maximum difference between water levels in the both sides of the structure has been considered in the present study.

#### THEORETICAL BACKGROUND

The theory of seepage has its origins in classical hydro-dynamics as the flow of an ideal fluid. Expressed as potential flow in an isotropic region, under the conditions of a fully saturated porous media, assuming that both pore fluid and soil grains are incompressible, the deformability of the soil skeleton can be neglected and Darcy's law holds, under these conditions, the general continuity equation in a two dimensional flow is as follows;

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0 \quad (1)$$

which gives the conventional Laplace equation.

$$(\partial^2 h / \partial x^2 + \partial^2 h / \partial y^2) = 0 \quad (2)$$

where  $h = y + P/w$

For potential flow in an isotropic homogeneous seepage domain when Darcy's law

$$v_n = -K_n \frac{\partial h}{\partial n} \quad (3)$$

is incorporated. Strictly, in more general case,

$$\frac{\partial}{\partial x} (K_x \frac{\partial h}{\partial x}) + \frac{\partial}{\partial y} (K_y \frac{\partial h}{\partial y}) = 0 \quad (4)$$

allows for arbitrary variation of the permeability coefficients in both space and direction.

Finite Element solution to this equation subject to conventional boundary conditions in two and three dimensions is trivial and the basic techniques are at least 20 years old (6). The solution to this equation is by means of fairly complex mathematics (1) which is only possible in simple seepage domains; by finite difference numerical methods, or by flow net sketching. Although it is possible to tackle complex shaped problems with the latter two, nonhomogeneous permeability presents the analyst with major problems.

The Finite Element method, in contrast, takes an energy approach, leading to the following integral:

$$I = \int_{\text{Vol. of seepage domain}} K_x \left( \frac{\partial h}{\partial x} \right)^2 + K_y \left( \frac{\partial h}{\partial y} \right)^2 d \text{ vol.} \quad (5)$$

which must be a minimum for a valid solution to the seepage pattern. Subdivision of the seepage domain into elements of finite size Fig.(1), enables the integration to be carried out simply, and the unknowns evaluated at key points or nodes where the elements "connect".

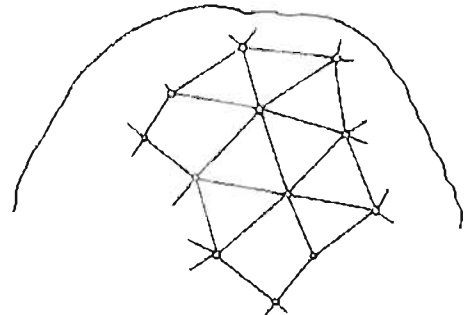


Fig. 1- Division of region into finite elements

The background to this is widely understood in structural and continuum mechanics (e.g. 7) and will not be further pursued here. For each element, the total head is approximated by a trial function (8):

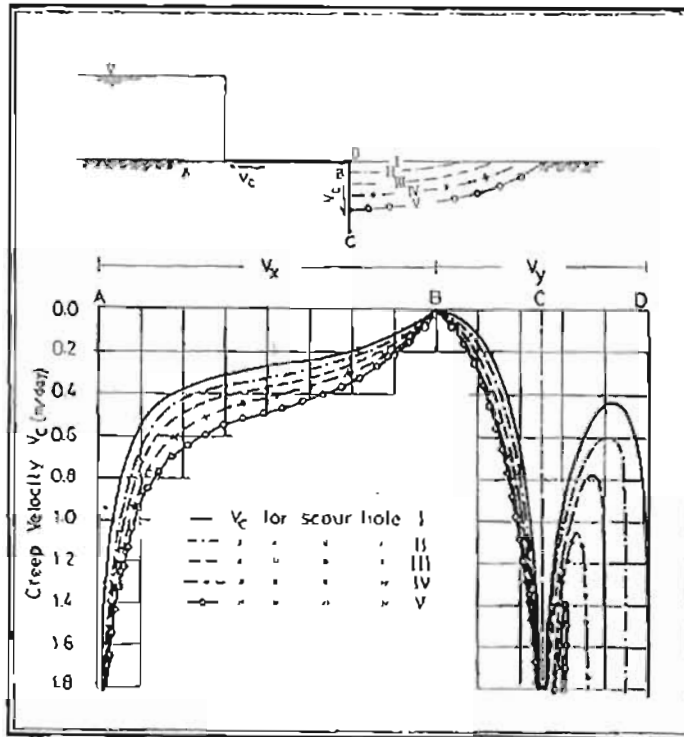


Fig.7 Illustration of the creep velocities according to the deformed beds

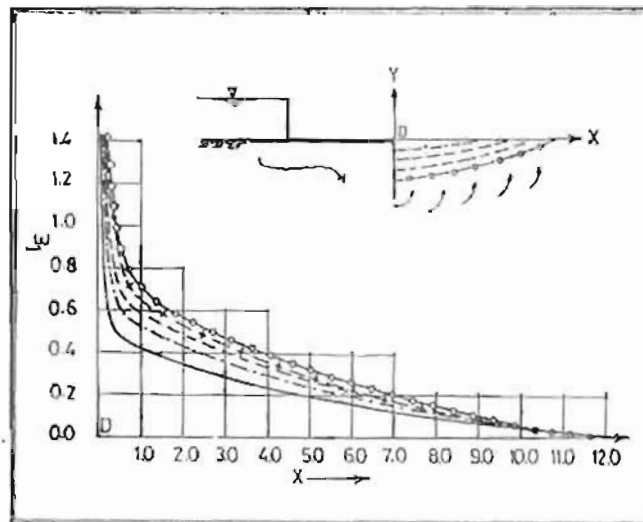


Fig.8 Variation of exit gradients according to the eroded beds

The floor creep velocities under the considered structure have been illustrated in Fig.(7). From this figure, it is observed that increasing the depression of the eroded bed increases the creep velocity. These velocities have infinite values at the points of singularity such as A, B, and C (see Fig.7).

Finally, the influence of the eroded beds on the hydraulic gradients at exit face have been illustrated graphically in Fig.(8). It is clear that increasing the eroded bed depression increases the value of the exit gradient, consequently the seepage flow rate increased from  $2.33 \text{ m}^3/\text{sec}$  for the first case to about  $4.0 \text{ m}^3/\text{sec}$  for the last case of the eroded beds ( $K_x = K_y = 1\text{m/day}$ ).

#### CONCLUSIONS: -

In this study the influence of the eroded beds on the characteristics of seepage underneath a weir has been accomplished numerically using the finite element technique. From the obtained results it is observed that, increasing the depression of the eroded beds increases the rate of seepage flow that seeps underneath the heading up structures, while decreases the acting uplift pressure. Consequently, creep velocity and exit gradients are increased causing a suitable conditions to the failure of the structure by the action of the piping phenomena.

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#### NOTATIONS

The following symbols have been adopted for use in this paper:

$g^w$  = unit weight of the fluid in the pores

$h$  = total head

$p$  = fluid pressure

$K_x, K_y$  = the permeability coefficients in  $x$  and  $y$  directions

$V_x, V_y$  = the velocity components in  $x$  and  $y$  directions, respectively

$V_n$  = velocity normal to the boundary

$\langle h \rangle$  = vector of nodal heads

$[K]$  = total flow matrix

$[N]$  = matrix contains the element shape function

$\langle Q \rangle$  = vector of nodal heads.