# PERFORMANCE EVALUATION OF DECISION FEEDBACK EQUALIZER UNDER MISMATCH

BY

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ABSTRACT. Transmitting digital data through dispersive channels is mainly affected by intersymbol interference (ISI). The decision feedback equalizer (DFE) was introduced as a suboptimum nonlinear processor to combat ISI for known channel characteristics.

In this paper, the performance of the DFE is invistigated for a mismatched channel. The channel characteristic under mismatched is allowed to vary with respect to a specified design channel characterstic for which the DFE is design. This investigation will allow us to examine the limits of mismatch for the channel performance is acceptable.

## I. INTRODUCTION

The operation and control of an electric power systems have become increasingly complex, as a result of the rapid increase in size of power networks, and the requirements for stable operation. Protection and control must be reliable, fast, and closely matched to the power system.

The improvement in component reliability has given us small computers,with this order of reliability they can be incorporated in many kinds of controls and data collections needed for power systems, the input to these computer must be in digital form. So, a highly reliable and high-speed digital data transmission is required to meet such demands for operation and control of power systems.

A primary limitation in any attempt to acheive a high transmission rate is the time dispersion suffered by the signal that is transmitted through the band-limited channel. In data transmission systems the time dispertion imparted on the transmitted signal results in an overlap in time between successive symbols (digits) which is called intersymbol interference (ISI) (1).

The other factor which affects the transmission of digital data is the additive noise introduced in the system.

The digital communication system can be modeled as shown in Fig. 1. (2).

The data sequence $\{a_k\}$ modulates a basic transmitting pulse filter x (t) at a rate 1/T. The total transmitted signal is

$$y(t) = \sum_{k=0}^{\infty} a_k \, x\,(t-kT) \qquad (1)$$

where T is the duration of the signalling interval and $a_k$ is the information symbol that is transmitted in the interval $kT < t < (k+1)\,T$, $K = 0, 1, \ldots$

The channel through which the signal transmitted is assumed linear, dispersive, and time invarient. It is characterized by the impulse response function c(t). Also the chanel noise is assumed additive .

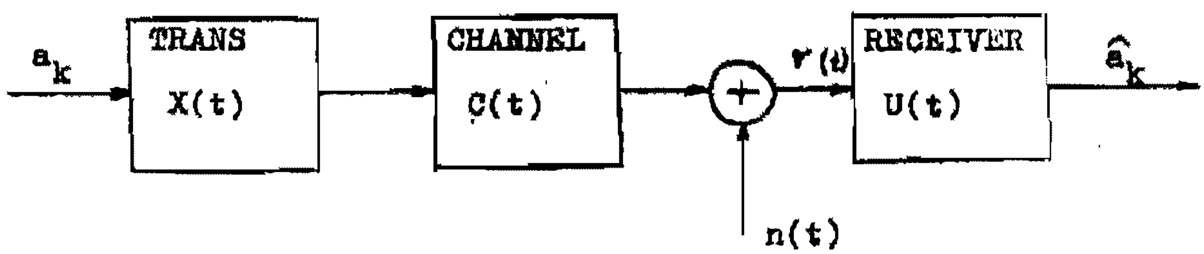


Fig. 1. Communication system model

The input to the receiver can be represented as

$$r(t) = \sum_{k=0}^{\infty} a_k \, h(t-kT) + n\,(t) \qquad (2)$$

where,

$$h(t) = \int_{-\infty}^{\infty} c(m)\, x\,(t-m)\, dm, \qquad (3)$$

and n(t) denotes the additive noise in the system.

The problem at the receiver is to detect the data sequence $\{a_k\}$ from the observation of r(t). So, the received waveform r(t) is processed by a receiver that can be linear or nonlinear and is optimized according to some measure of performance. Thus, the receiver is designed to combat the noise and intersymbol interference.

The research effort (1), (5) , was mainly carried on assuming a receiver structure which consisted of a linear signal processor followed by a thresholding device, reffered to as Linear Equalizer (LE).

Mostly the used measure of performance is the mean square of error (MSE). The optimum receiver with minimum MSE is obtained under the constraint that the ISI was completely eliminated at the sampling instants, the zero forcing condition (ZF), and without that constraint (2), (4).

Using the probability of error as a performance measure for obtaining optimum LE was found (6),(7). But determining the system parameters requires considerable numerical effort.

The obtained LE processor is a combination of a matched filter and a tapped delay line.

The inability of LE to cope with severe ISI has directed the research to nonlinear techniques (2).

Nonlinear processor are based on attempts to implement either a maximum likelihood sequence estimation (8) or a maximum aposteriori probability

(MAP) (9) decision rule. It was shown that nonlinear processor provide significant improvement over the simple linear types (2).

There is a much interest in finding suboptimum nonlinear processor with significant performance advantages over the linear equalizer without the complexity of the nonlinear optimum solutions. This leads us to the decision feedback equalizer (10), (11) which is illustrated in Fig. 2. It consists of two parts a feedforward part (FFF) and feed back part (FBF). The feedforward part is a matched filter plus a tapped delay line with taps spaced at the symbol interval T, and has at its input the sequence $(r_k)$. The feedback part is also a tapped delayline, whose input are decisions on previously detected symbols. In this case the tapped delay line of forward filter processes the signal to reduce the (ISI) of all future symbols, while the feedback filter subtracts the (ISI) due to all past symbols.

The channel characteristics are not always available to the designer, also it may be time varying. To overcome such problems adaptive processors are investigated (2), (10), (11), (16), (17),(18).

But this adaptation will introduce some new elements which in turn increase the complexity of the receiver used. Also, we can expect that this new elements will affect its performance.

In our study we shall find the limits of variation of channel characteristic for which the performance of DFE is acceptable, without use of system adaptation.

The parameters for optimum DFE for known channel characteristic are reviewed (12) in section 2. The mean square error for mismatched case is then derived, section 3. Finally an application of the derived results

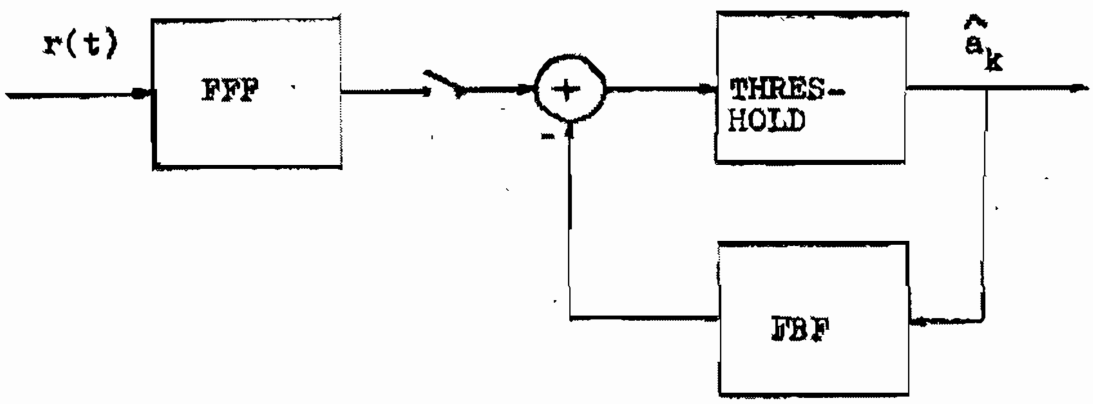


Fig. 2. Decision Feedback equalizer

using a coaxial cable channel is examined.

II- MINIMUM MSE OF DFE FOR MATCHED CASE

In designing DFE, it is assumed that the channel frequency characterstic is known for the designer so, we will have a matched DFE. This case of observations will be denoted a matched case. If the channel frequency characteristic is changed while the DFE remain the same, then we will have a mismatched case of observation.

We shall use in our study a special structure of decision feedback equalizer different from the classical one illustrated at Fig. 3. (12). It was shown that this is equivelent to the DFE shown in Fig. 2. (12). The advantage of using this alternate structure is that the forward filter is independent on the number of feedback taps, and is infact the otpimum linear equalizer. So it

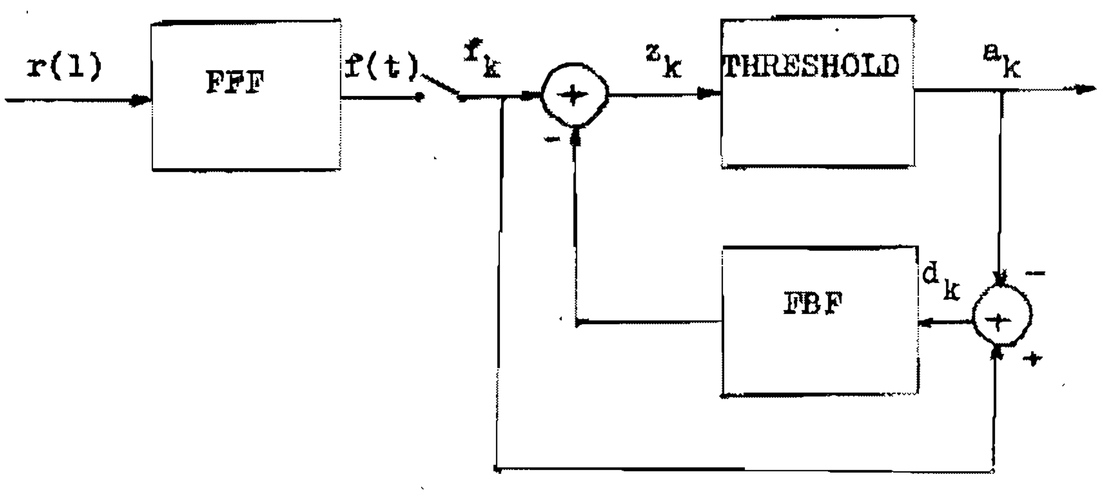


Fig. 3. Decision feedback equalizer

will be easy to make a comparison between DFE and LE.

Using the MSE as a performance criterion for obtaining an optimum receiver, and assuming that the past N decisions are correct (12).

The error between the input to the threshold device and the input information sequence is denoted as $e_k$ , where,

$$e_k = d_k - g_k * \hat{d}_k \qquad (4)$$

$$d_k = f_k - a_k \qquad (5)$$

$$\hat{d}_k = d_k + (a_k - \hat{a}_k) \qquad (6)$$

and $g_k$ is the tap gains of the feedback filter.

The performance measure is ;

$$MSE = E\,(e_k^{\,2}) = T \int_{-\frac{1}{2}T}^{\frac{1}{2}T} R_e\,(f)\,df \qquad (7)$$

where,

$$R_e(f) = \left| 1 - G\,(f) \right|^2 \; R_d\,(f) \qquad (8)$$

$$R_d(f) = \sigma_a^2 \; \left| \Phi(f) - 1 \right|^2 + \frac{1}{T} \sum_n N_n(f) \; \left| U_n(f) \right|^2 \qquad (9)$$

$$\Phi(f) = \frac{1}{T} \sum_n x_n\,(f)\; C_n(f)\; U_n(f) \qquad (10)$$

$$G(f) = \sum_{k=1}^{N} g_k \,\exp\,(-j2\pi kfT) \qquad (11)$$

and for brevity we have used $N_n\,(f)$ for $N(f+n/T)$, etc.

All previous and next sums are from $-\infty$ to $+\infty$, if not declared.

The input sequence ..., $a_{k-1}$, $a_k$, $a_{k+1}$, ..are drawn from the set $-$ M+1 , $-$ (M+3), ... , $-1$, 1, ... , (M+1) . They are assumed to be statistically independent with variance given by ,

$$E\,(a_k\, a_e) = \sigma_a^2 \; \delta_{k-e} \qquad (12)$$

where

$$\sigma_a^2 = (M^2-1)/3. \qquad (13)$$

E denotes expecations, and $\delta_n$ is the kronecker delta.

Thus, the signal to noise ratio (S/N) at the input of the DFE will be given by ,

$$\frac{S}{N} = \frac{P}{N0/2} = \frac{\sigma_a^2 \int_{-\infty}^{\infty} h^2(t)\;dt}{N0/2} \qquad (14)$$

where the additive noise n(t) is white with spectral density N(f) and assuming it is equal N0/2, and P is the average signal energy per sampling peroid T.

For minimzing the MSE, it was found (12) that the optimum forward flter is given by,

$$U(f) = \frac{x^*(f)\;C^*(f)}{N\,(f)} \; \frac{\sigma_a^2}{1 + \sigma_a^2 \,\frac{1}{T} \sum_n \frac{|x_n(f)\;C_n(f)|^2}{N_n(f)}} \qquad (15)$$

and the optimum backward filter is given by,

$$(g_1\,,\, g_2\,,\, \ldots\,,\, g_N)^T = B^{-1}\,(b_1,\, b_2,\, \ldots.\, b_N)^T. \qquad (16)$$

where, B is an N x N matrix with entreis

$$(B)_{m,n} = b\,(m-n) \qquad 1 \leq m,\; n \leq N \qquad (17)$$

and

$$b_k = T \int_{-\frac{1}{2}T}^{\frac{1}{2}T} \frac{\sigma_a^2 \;\exp\,(-j2\pi fkT)}{1+ \sigma_a^2 \,\frac{1}{T} \sum_n \frac{|x_n(f)\;C_n(f)|^2}{N_n(f)}}\; df \qquad (18)$$

The minimum MSE is given by,

$$MMSE = b_o - (b_1, b_2, \ldots, b_N)^T B^{-1} (b_1, b_2, \ldots b_N)^T \quad (19)$$

The optimum forward filter given by Eq. (15) can be realized with a matched filter followed by a tapped delay line with transfer function,

$$T_u(f) = \sum_k b_k \exp(-j2\pi fkT) \quad (20)$$

and the tap gains are given by Eq. (18).

III- THE MSE OF DFE FOR MISMATCHED CASE.

In this section we shall find an expression for the MSE for mismatched observation case. The DFE will be optimized for certain design channel response. Then we shall find the dependence of the MSE when the channel response is changed while the DFE remains the same as optimized for the specified design channel.

The idea of analyzing a system performance for mismatched observation case appeared in (13) when analyzing the performance of sequential processors, and in (14), (15) when analyzing the mismatched performance of the likelihood ratio processors.

Now we shall use the subscript "d" for the specified channel to which the DFE is designed, and the subscript "a" for the actual channel through which the actual data, to be processed, is transmitted.

From Eq. (10). We can write for the specified design channel,

$$\Phi_d(f) = \frac{1}{T} \sum_n X_n(f) \quad C_{n\,d}(f) \quad U_n(f) \,, \tag{21}$$

and for the actual channel,

$$\Phi_a(f) = \frac{1}{T} \sum_n X_n(f)\ C_{na}(f) \quad U_n(f) \tag{22}$$

For the mismatched case $R_d^{`}(f)$ will be given by,

$$R_d^{`}(f) = \sigma_a^2 \left|\Phi_a(f) - 1\right|^2 + \frac{1}{T} \sum_n N_n(f) \left| U_n(f)\right|^2 \tag{23}$$

The MSE is

$$E\ (e^2) = T \int_{-\frac{1}{2}T}^{\frac{1}{2}T} \left|1 - G(f)\right|^2 \ R_d^{`}(f) \quad df \tag{24}$$

An alternative form of Eq. (24) can be obtained using Eq. (4) as follows,

$$E\ (e^2) = E\left(\left|d_k - \sum_{n=1}^{N} g_n \cdot d_{k-n}\right|^2\right)$$

$$= E\left(\left|\sum_{n=0}^{N} g_n^{`} \cdot d_{k-n}\right|^2\right)$$

$$E\ (e_k^2) = \sum_{n=0}^{N} \sum_{i=0}^{N} g_n^{`} g_i^{`} \quad E\ (d_{k-n} \quad d^*_{k-i})$$

$$= \sum_{n=0}^{N} \sum_{i=0}^{N} g_n^{`} g_i^{`} \quad W_{(n-i)} \tag{25}$$

where $g_n^{`}$ equals $(\delta_n - g_n)$ , and $W_{(k)}$ is the sampled correlation function of the sequence $(d_k)$ and they are related to the power spectrum $R_d^{`}(f)$ by

$$W_{(k)} = T \int_{-\frac{1}{2}T}^{\frac{1}{2}T} R_d^{`}(f) \exp\ (-j\ 2\pi fkT)\ df\ .. \tag{26}$$

After simple arrangement of (25) , the MSE can be written in the form,

$$MSE = W_o - 2GD^T + GAG^T \tag{27}$$

where the matrices D, and G are given by

$$D = (W_1, W_2, \dots W_N) \quad , \tag{28}$$

$$G = (g_1, g_2, \dots g_N) \quad , \tag{29}$$

and A is amatrix NxN with entreis,

$$(A)_{m,n} = W_{(m-n)} \qquad , 1 \leq m , n \leq N \tag{30}$$

The expression obtained in Eq. (27) express the mean square error for the mismatched case, i.e the channel characteristic differ from that one for which the DFE is optimized.

The values of $g_1, g_2 \dots$ are obtained from Eq. (16).

## IV APPLICATION

We shall investigate the results obtained in section III. using as an example the coaxial cable channel. The amplitude frequencey characteristic of this channel is given by (12).

$$|C(f)| = \left[ \exp \left(- Q \, |f| \right) \right]^{1/2} \tag{31}$$

where, Q is depending on the repeater spacing.

Without loss of generality the transmitter is assumed an ideal low-pass filter with frequency characteristic $x(f) = 1$.

The number of feedback taps (N) of DFE is chosen to be equal 6.

In our investigation we shall consider the attenuation of the channel at 1/2 T equal 60 dB for matched case and denote it Cd.

For mismatch the value of the channel attenuation at ½ T will be denoted by Ca, and will takes different values arround Cd.

A plot of mean square of error of DFE and LE for mismatch versus Ca is shown in Fig. 1., Fig. 2., and Fig. 3., for different signal to noise ratio 20, 40, and 60 dB, respectively. In the same figure a plot of mean square of error of DFE and LE for matched case. At Fig. 4. and 5., a comparison between DFE and LE for differen signal to noise ratio at constant channel attenuation (Cd = 60 dB), and for different Cd keeping signal to noise ratio fixed at 40 dB.

## V. CONCLUSIONS

Using the coaxial cable as an example of a highly dispersive channel, it is found that the performance of decision feedback equalizer under mismatch is quite well for certain range of channel attenuation variation, but this range is inversely proporational to the signal -to-noise ratio.

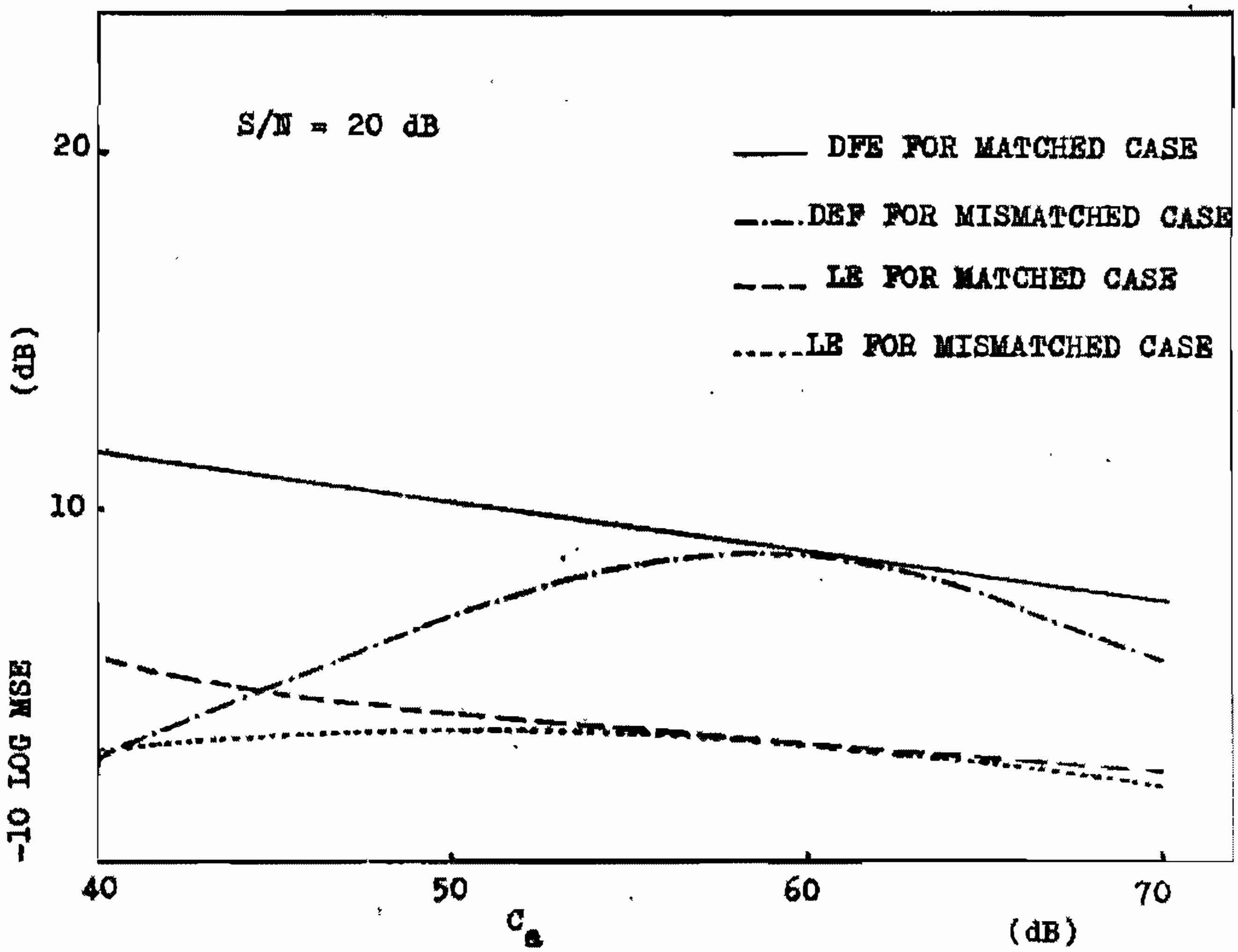


Fig. 1 : Performance of DFE and LE under mismatch.

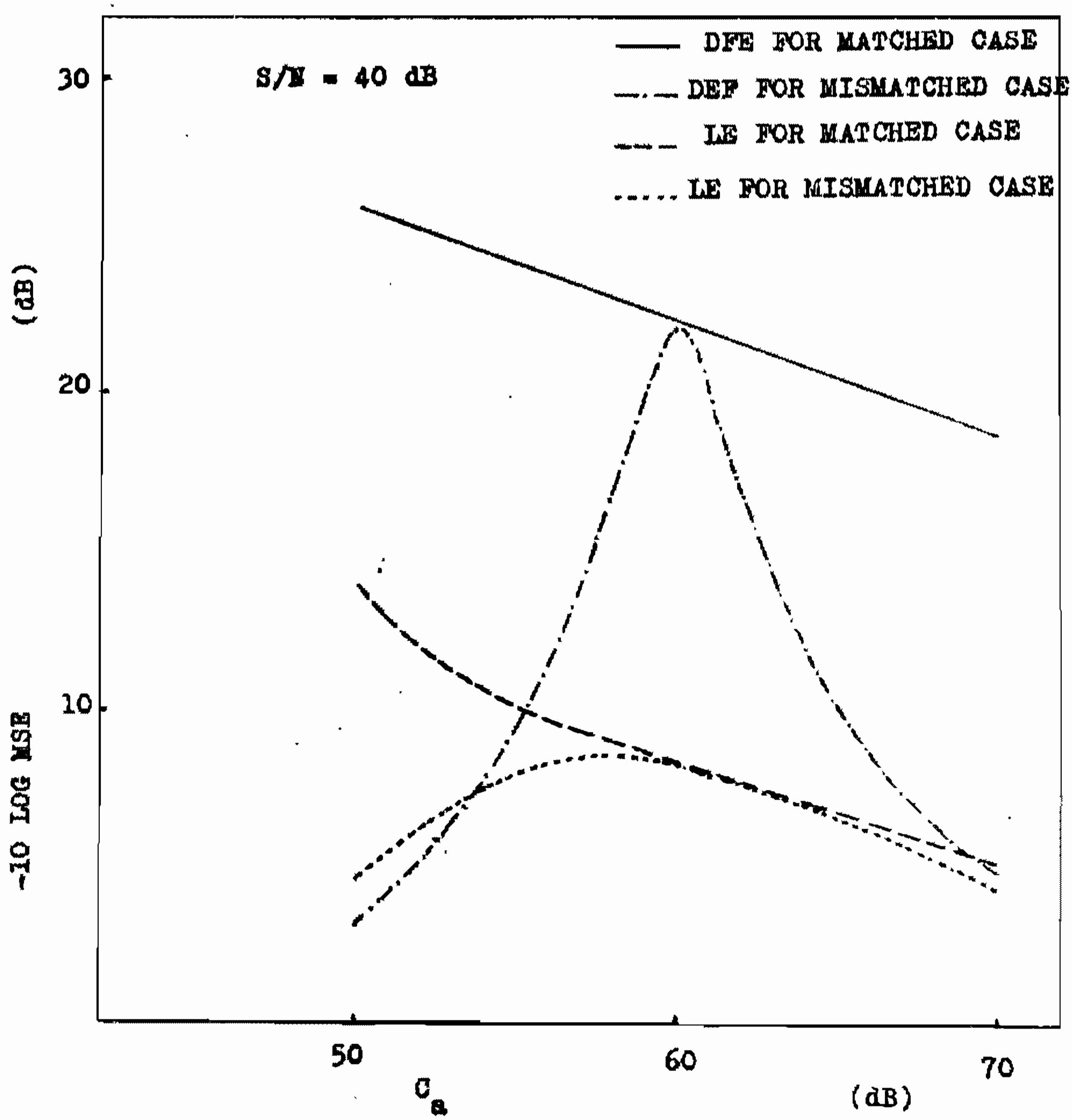


Fig. 2 : Performance of DFE andLE under mismatch.

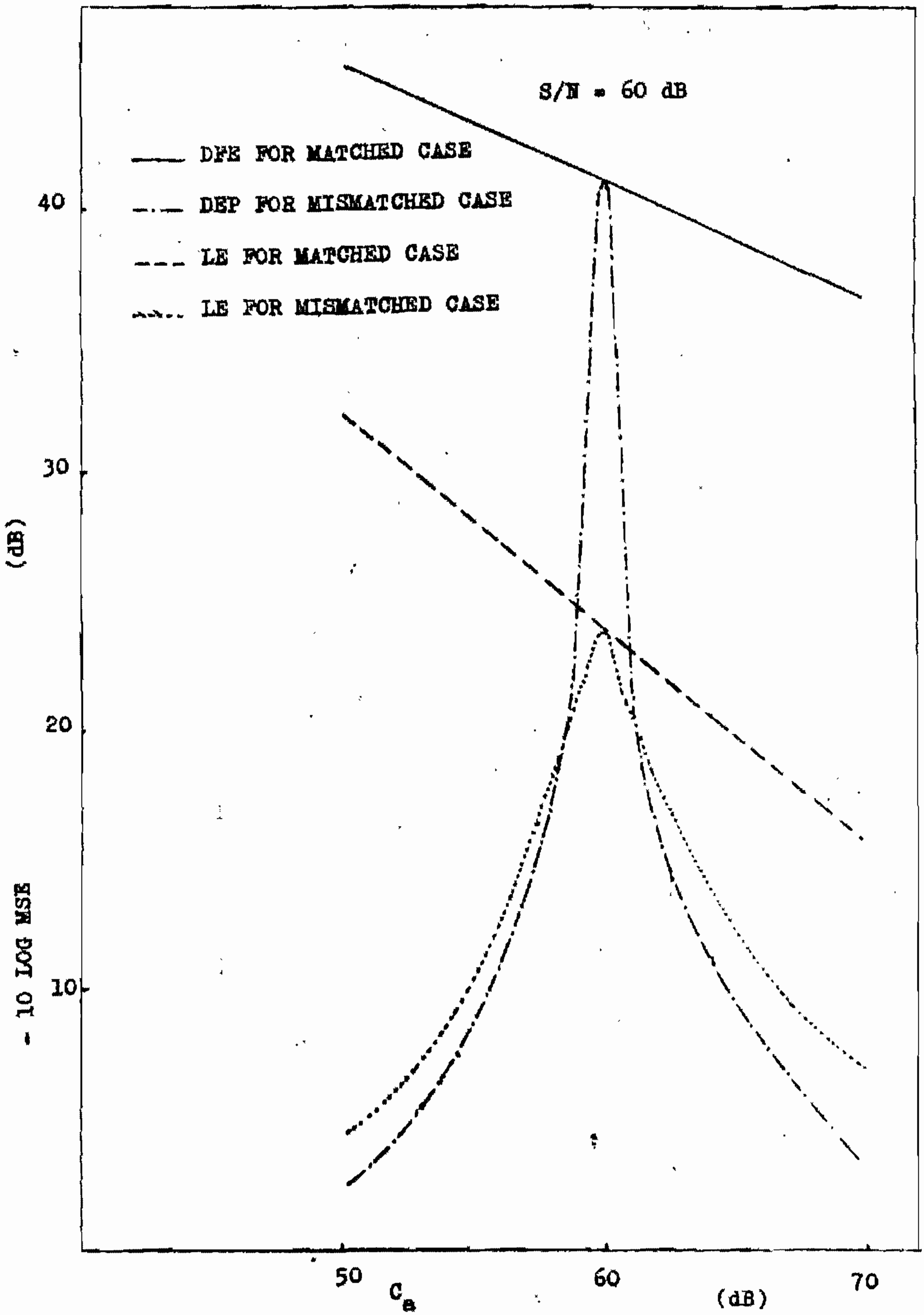


Fig. 3 : Performance of DFE and LE under mismatch.

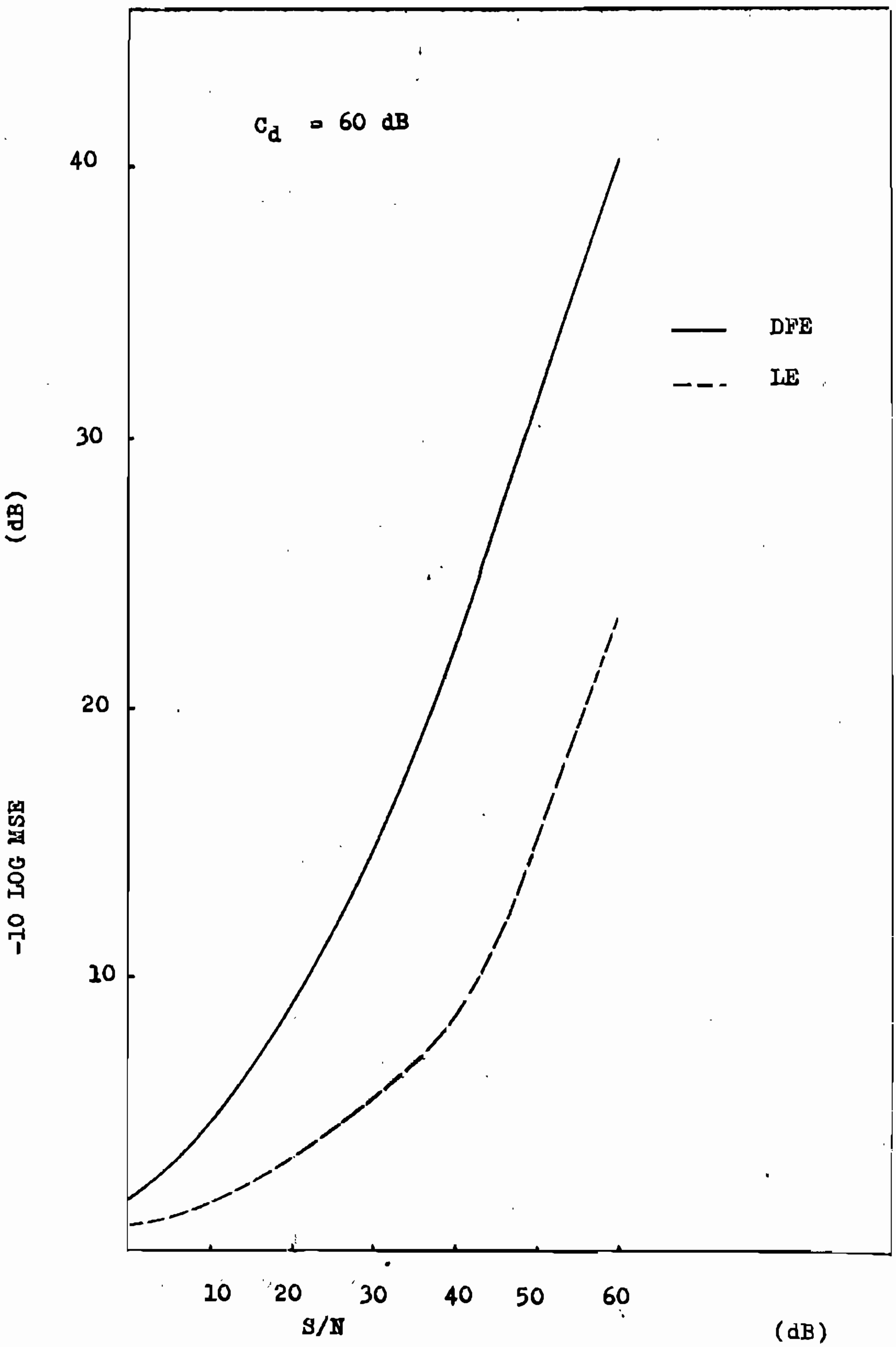


Fig. 4 : Comparison of DFE and LE for matched case.

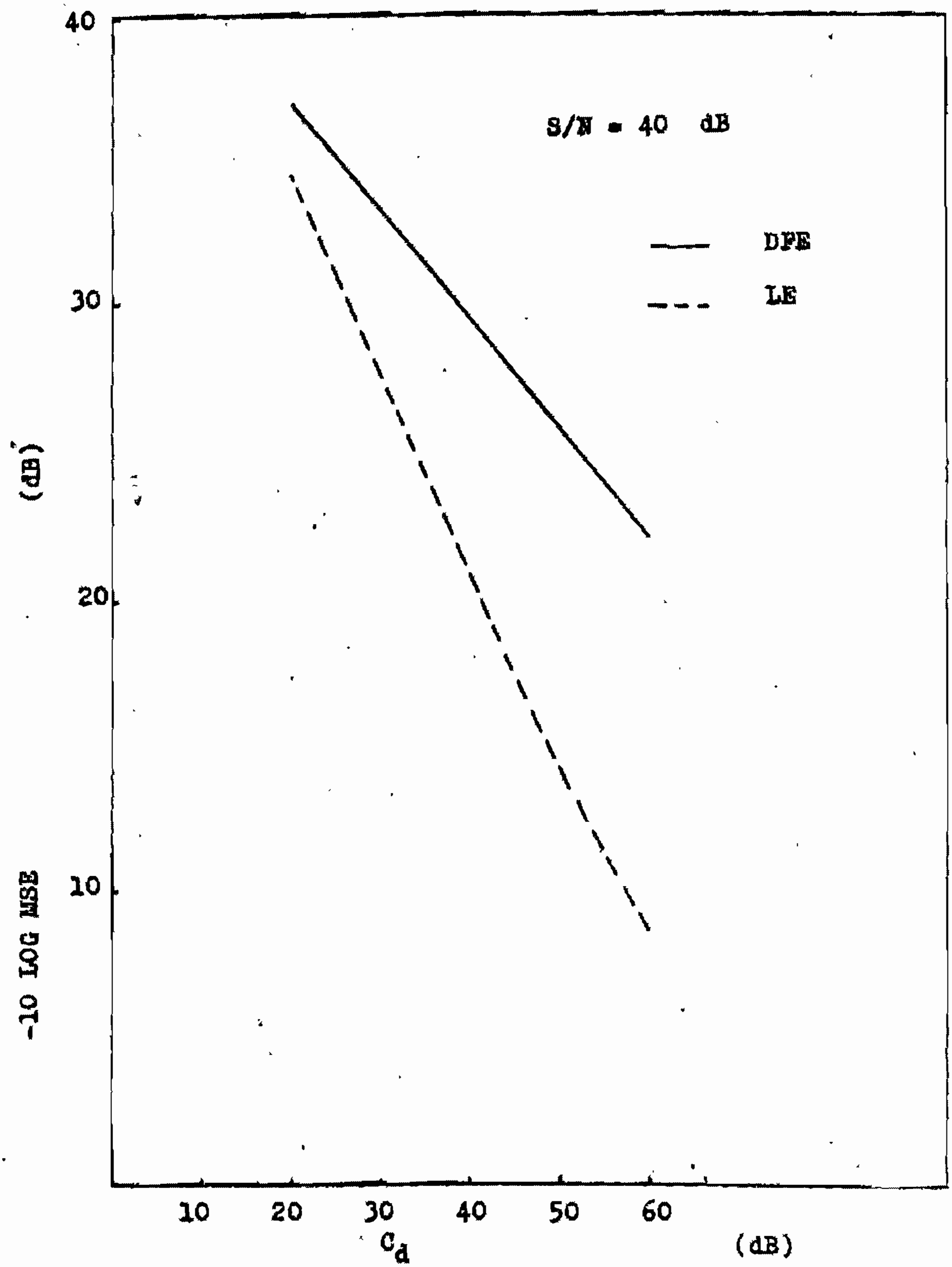


Fig.5 : Comparison of DFE and LE for matched case.

If we compare the meansquare error of DFE under mismatch and LE for matched case, we can see that the following variation of channel attenuation , is acceptable, for signal-to-noise ratio equals 20 dB, the permisable channel attenuation variation ( $\Delta C$) is approximately $\pm$ 15 dB, for S/N = 40 dB, $\Delta C$ from -5 to + 9 dB, and for S/N = 60 dB , $\Delta C$ is equal $\pm$ 1 dB.

Also it is seen that the DFE is advantagous over the LE in the mismatched case, when the actual channel attenuation is near to the design value. But far from this zone, the performance of DFE and LE are approxisimately the same.

It is shown also the superiority of DFE over LE for matched Case. It is found that the MMSE for DFE and LE decreases with the increase of signal-to-noise ratio and with the decrease of channel attenuation, but the rate of decreasing of DFE is more greater than LE.

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