

## ULTIMATE SHEAR STRENGTH OF PLATE GIRDERS WITH REINFORCED WEB OPENINGS

مقاومة القص القصوى للكمرات اللوحية ذات الفتحات المقواه في العصب

By

Ahmed Badr

Assoc. Prof. of Structural Eng. Faculty of Eng. - Mansoura University



في هذا البحث تم دراسة وحساب مقاومة القص القصوى للكمرات اللوحية ذات الفتحات المستديرة أو المستطيلة في العصب. وفي هذه الدراسة تم حساب الحمل المرين الحرج وحمل الشد في عصب الكمرة عند المرحلة التالية للحمل الحرج وكذلك الأحمال والإجهادات على شفة الكمرة اللوحية وأيضاً الأحمال والإجهادات الناتجة على تقويات العصب حول الفتحات. وقد تم استخدام نظرية العناصر الدقيقة لحساب الحمل المرين الحرج والمقاومة القصوى لهذه الكمرات اللوحية ذات الفتحات المستديرة أو المستطيلة. وفي النهاية تم مقارنة النتائج النظرية التي تم الحصول عليها في هذا البحث مع النتائج المعملية السابقة على نماذج لكمرات لوحية. وأعطت المقارنة نتائج جيدة مما يؤكد صحة ما تم دراسته.

### ABSTRACT

A theoretical method is proposed for calculating the ultimate shear capacity of plate girders with reinforced web holes as the sum of four contributions, viz.

- (i) the elastic critical load,
- (ii) the load carried by the membrane tension in the web in the post-critical stage,
- (iii) the load carried by the flange and
- (iv) the load carried by the reinforcement.

Approximate formulae based on a finite element analysis are suggested for calculating the elastic critical loads in shear. The membrane stresses are calculated by using the von Mises yield criterion; the contributions of the flanges and of the reinforcement are obtained from their plastic moments. Ultimate load tests on plate girder models presented previously are compared with the theoretical prediction using the analytical model described in the paper.

#### C. 4 Ahmed Badr

From a consideration of virtual work, the ultimate shear can be evaluated in a manner similar to the one derived by Porter et al [1]:

$$V_{ult} = 2C \cdot \sigma_t^y \cdot t \sin^2 \theta + \sigma_t^y \cdot t \cdot h (\cot \theta - \cot \theta_d) \sin^2 \theta - \sigma_t^y \cdot t \cdot d \left(1 - \frac{b_c}{d}\right) \sin \theta + (\tau_{cr}) \cdot h \cdot t \quad (6)$$

in which;

- $V_{ult}$  ultimate shear load for perforated web,
- $C$  distance between the flange hinges,
- $\theta$  angle of inclination of the tensile membrane stress, and
- $\theta_d$  angle of inclination of the panel diagonal.

The value of  $C$  is obtained from  $\frac{2}{\sin \theta} \sqrt{\frac{M_p}{\sigma_t^y \cdot t}}$  and  $\sigma_t^y$  is evaluated by applying the Von Mises criterion :

$$\sigma_t^y = -\frac{3}{2} (\tau_{cr}) \sin \theta + \sqrt{\sigma_{yw}^2 + (\tau_{cr})^2 \left[ \left( \frac{1}{2} \sin 2\theta \right)^2 - 3 \right]} \quad (7)$$

in which ,

- $\sigma_{yw}$  yield stress of web plate.

The values of  $V_{ult}$  obtained from equation (6) are dependent on the chosen  $\theta$  ; the maximum value of  $V_{ult}$  is obtained by trial and error.

#### 2. PLATE GERDERS WITH REINFORCED RECTANGULAR HOLES

It is usual to reinforce rectangular holes by welding flat plates to the web, above and below the opening (Fig. 4). The reinforcement should project beyond the edges of the hole for a length sufficient to allow the full development of the tension field; this anchorage length ( $l$ ) will, therefore, depend on the dimensions of the opening, a web having a deeper opening requiring more anchorage length. Tests conducted previously [4,10] suggest that the minimum length of the reinforcement,  $L_r$ , should be  $1.5 b_0$  or  $\sqrt{b_0^2 + d_0^2}$  whichever is larger in order that the strength lost by the hole can be fully recovered; in addition, the reinforcement should have sufficient stiffness to resist the membrane tension.

Based on extensive finite element studies, the following minimum reinforcement requirement was found to be essential in order that the buckling coefficient for the perforated web would be at least equal to that of an unperforated web:

$$\left( \frac{t_r}{t} \right)^2 \cdot \left( \frac{w_r}{h} \right) \geq 2.76 \sqrt{\frac{b_0 \cdot d_0}{b \cdot h}} \quad (8)$$

in which;

- $t_r$  thickness of reinforcement (steel strips),
- $b_0$  width of rectangular opening, and
- $d_0$  depth of rectangular opening.

The second and third stages in the incremental loading for the plate girder with rectangular openings are similar to the case of circular openings. Four hinges are formed on the

reinforcement as shown in Fig. (5). By the method of Virtual work, the following equation for the ultimate shear is obtained :

$$V_{ult} = 2c\sigma_t^y \cdot t \sin^2 \theta + \sigma_t^y \cdot t \cdot h (\cot \theta - \cot \theta_0) \sin^2 \theta - \sigma_t^y \cdot t \sqrt{b_0^2 + d_0^2} \sin(\alpha + \theta) \sin \theta + 2c_r \cdot \sigma_t^y \cdot t \cdot \sin^2 \theta + (\tau_{cr}) \cdot h \cdot t \quad (9)$$

in which;

$\alpha$  angle of inclination of the diagonal of the rectangular opening, and  
 $c_r$  distance between hinges on the reinforcement.

In the above equation  $c$ ,  $\sigma_t^y$  are calculated as previously indicated for circular openings. If the reinforcement had adequate end fixity, then the hinge distance  $c_r$  is obtained from

$$c_r = \frac{2}{\sin \theta} \sqrt{\frac{M_{pr}}{\sigma_t^y \cdot t}} \quad (10)$$

### 3. EXPERIMENTAL INVESTIGATION [4]

Experimental work carried out previously [4] has shown that in spite of the inherent problems associated with scale effects and welding stresses, small scale models can be employed satisfactorily to assess the behaviour of plated structures. Model girders made of 1 mm webs were therefore used to carry out tests on plate girders containing reinforced web holes. Series 1 consisted of 8 shear panels containing circular holes and Series 2, of 12 shear panels with rectangular holes. The parameters varied were : (a) web aspect ratio (b/h) ... (b) web slenderness (h/t) ... (c) sizes of openings and ... (d) the size of reinforcement.

Figure (6) gives the design details of Series 1 girders; figure (7) gives similar particulars of Series 2 girders.

Each of the test girders consisted of two test panels; only one panel was tested at a time. The panel not under test was temporarily stiffened by clamping two stiffeners to its web in order to prevent any distortion occurring on it, while the other panel was being tested. This procedure was repeated with the second panel.

Table 1. Gives the measured loads from these tests and compare them with the predicted values of ultimate shear, using reinforced opening are generally greater than those obtained with unperforated webs.

The loads predicted are seen to be consistently safe and satisfactory. The mean value of, predicted ultimate load / previously experimental ultimate load [4], for the 6 tests in series 1 (with circular openings) was 0.895 with a standard deviation of 0.037; the corresponding mean value for 10 tests in series 2 with rectangular openings was 0.931 with a standard deviation of 0.055 (see Table 1).

### CONCLUSION

The paper presents methods of predicting the ultimate shear of plate girders containing reinforced rectangular and circular openings. The theory accounts for the buckling load of the web, the post-buckling membrane stress developed in the flanges and in the reinforcement. Comparison of result on model girders confirm the validity of the theory.

**REFERENCES**

- [1] Porter, D.M., Rockey, K.C. and Evans, H.R. " the collapse behaviour of plate girders loaded in shear " *the structural engineer*, Vol. 53, No. 8, august 1975, pp. 313-325.
- [2] Rockey, K.C. , Anderson, R.G. and Cheung, Y.K. " The behaviour of square shear webs having a circular hole." *Proceedings of a symposium on thin Walled Structures*, Crosby - Lockwood & Sons, 1967, pp. 148-169
- [3] Zienkiewicz, O.C. and Cheung, Y. K. " The Finite Element Method for the analysis of elastic isotropic and orthotropic slabs ". *Proceedings of the Institution of Civil Engineers*, Vol. 28, Aug. 1964, pp. 471-488.
- [4] Adoriso, D. and Narayanan, R. " Model studies on plate girders". *Journal of Strain Analysis*. Institution of Mechanical Engineers, London. Vol. 38, Oct. 1983.
- [5] Narayanan, R. and Rockey, K. C. " Ultimate capacity of plate girders with webs containing circular cut-out" *Proceedings of the Institution of Civil Engineers*, London, Part 2, Vol 72, Sept. 1981, pp. 845-862.
- [6] Badr, A. " Design of steel beams with multiple openings. " *Thesis submitted for the degree of master of science*, El-Mansoura Univ. 1980.
- [7] Abd-Rabou, S Abu-Mosallm, Y. and Badr, A. " Failure tests of steel beams with Eccentric web openings. " *Mansoura Eng. Journal (MEJ)*, Vol 20, No. 3, Sept. 1995.
- [8] Badr, A. and EL.Banna, M. " Plastic behaviour of steel beams with mid-depth Equi-spaced Rectangular openings." *Ain Shams Univ. Eng. Bulletin*, vol . 25 No 2 August 1990
- [9] Badr, A. and El-Banna, M. " Plastic loads and resistance factors of steel beams with stiffened holes. " *Mansoura Eng. Journal (MEJ)*, Vol. 15, No. 2, Dec. 1990.
- [10] T S. Arda, & G. Bayramoglu " An experimental study on butt welded steel beams with half NPL chords and rectangular openings." *Istanbul Technical Univ. Press*, 1993, Istanbul, Turkey.
- [11] Grayson, W.R. " *Computer Program User Manual for NASP (Mark 2) and NASPLOT* " *Simon Engineering Laboratories, University of Manchester*, Aug 1981

**Table 1**

PREVIOUS EXPERIMENTAL VALUES OF ULTIMATE LOAD COMPARED WITH PREDICTED VALUES

| panel designation  | predicted ultimate load (KN) | Experimental ultimate load [4] (KN) | Predicted load / Experimental load [4] |
|--|------------------------------|-------------------------------------|--|
| <b>Series 1 :</b>  |                              |                                     |  |
| TCP1A  | 31.8                         | 33.1                                | ***                                    |
| TCP1B  | 33.5                         | 36.2                                | 0.925                                  |
| TCP2A  | 39.1                         | 43.5                                | 0.899                                  |
| TCP2B  | 49.6                         | 49.6                                | 0.939                                  |
| TCP3A  | 43.1                         | 53                                  | ***                                    |
| TCP3B  | 46                           | 55.8                                | 0.824                                  |
| TCP4A  | 54.4                         | 61.8                                | 0.88                                   |
| TCP4B  | 68.7                         | 75.9                                | 0.905                                  |
| Mean value of (Predicted load)/(Experimental load) for 6 tests = 0.895<br>standard deviation = 0.037 |                              |                                     |  |
| <b>SERIES 2 :</b>  |                              |                                     |  |
| GP1A   | 43.2                         | 46.3                                | ***                                    |
| GP1B   | 43.3                         | 46                                  | 0.941                                  |
| GP2A   | 48.3                         | 55.5                                | 0.869                                  |
| GP2B   | 48.8                         | 55                                  | 0.888                                  |
| GP3A   | 46.6                         | 46.3                                | 1.007                                  |
| GP3B   | 44.2                         | 46.4                                | 0.953                                  |
| GP4A   | 44.3                         | 48                                  | 0.092                                  |
| GP4B   | 43.1                         | 45.5                                | 0.947                                  |
| TCP5A  | 34.3                         | 34.3                                | ***                                    |
| TCP5B  | 39.5                         | 39.5                                | 1.04                                   |
| TCP6A  | 33.5                         | 33.5                                | 0.868                                  |
| TCP6B  | 33.8                         | 33.8                                | 0.895                                  |
| Mean value of (Predicted load /experimental load) for 10 tests = 0.931<br>standard deviation = 0.055 |                              |                                     |  |

\*\*\* These are control tests carried out on pannel without opening.  
It will be seen that the collepce loads of models having reinforced openings are at least equal to those with unperforated webs.

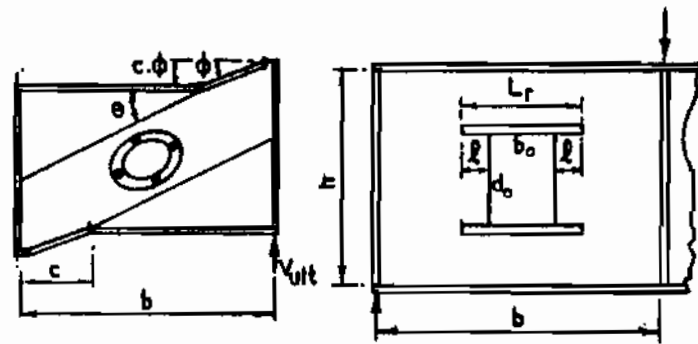
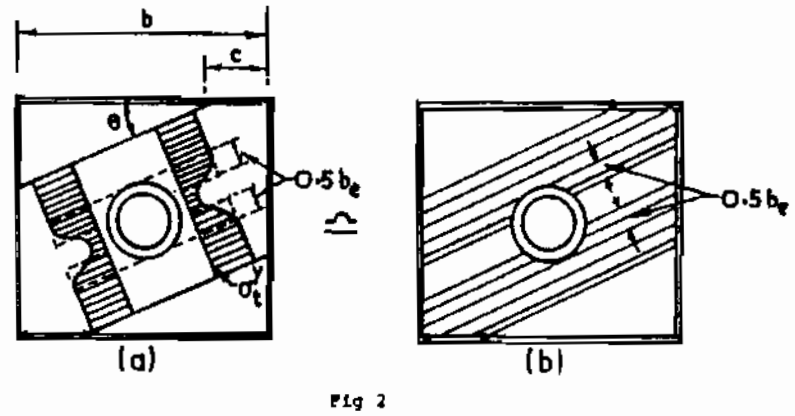
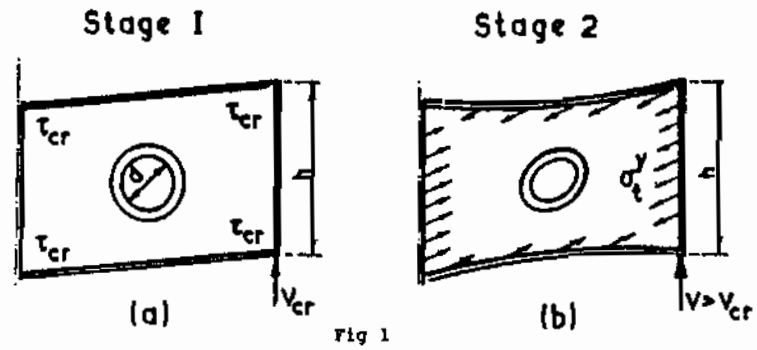


Fig 3 Collapse state

Fig 4 Flat plate reinforcement

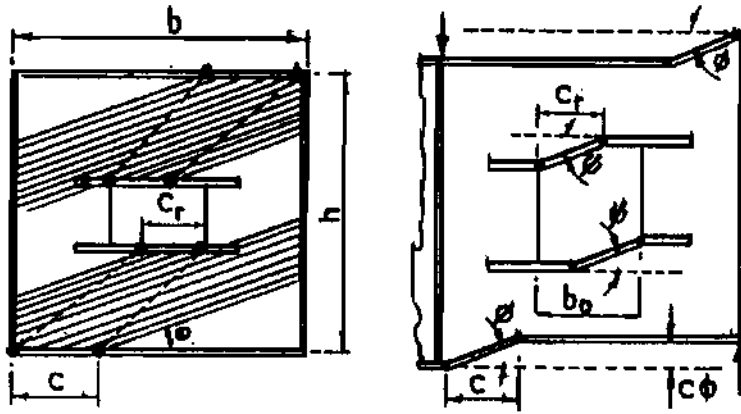


Fig 5

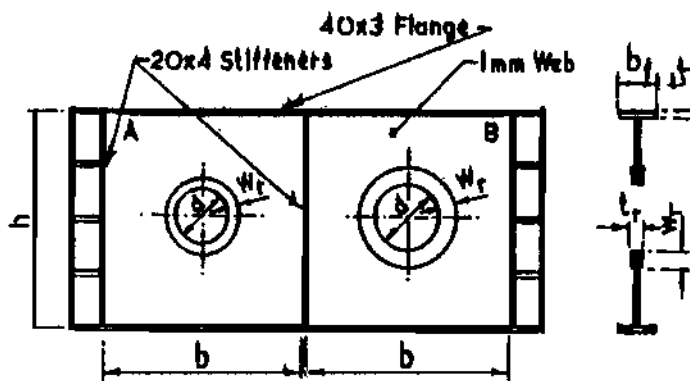


Fig 6 Design details of test specimens - Series 1 (4)

| Girder No. | Flange $b_f \times t_f$ | Web $b \times h$ | CUTOUT IN PANEL A |               | CUTOUT IN PANEL B |               |
|------------|-------------------------|------------------|-------------------|---------------|-------------------|---------------|
|            |                         |                  | Diameter          | Reinforcement | Diameter          | Reinforcement |
| TCP 1      | 40 x 3                  | 375x250          | 0                 | =             | 55                | 11 x 5        |
| TCP 2      | 40 x 3                  | 375x250          | 85                | 17 x 5        | 125               | 25 x 5        |
| TCP 3      | 40 x 3                  | 360x360          | 0                 | =             | 80                | 16 x 5        |
| TCP 4      | 40 x 3                  | 360x360          | 122               | 25 x 5        | 180               | 37 x 5        |

Notes: All dimensions are in mm. All panels are made of Grade 43 steel. All webs are cut from single sheet of rolling. All flanges are cut from a single batch of rolling.

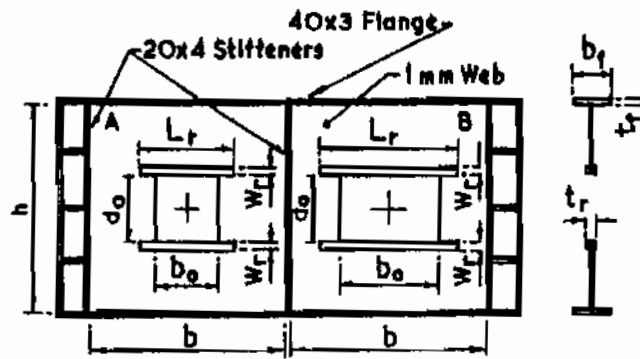


Fig 7 Design details of test specimens - Series 2

| Girder No. | Flange $b_f \times t_f$ | Web $b \times h$ | CUTOUT IN PANEL A |               | CUTOUT IN PANEL B |               |
|------------|-------------------------|------------------|-------------------|---------------|-------------------|---------------|
|            |                         |                  | Size              | Reinforcement | Size              | Reinforcement |
| GP 1       | 40 x 3                  | 300x300          | 0                 | "             | 100x90            | 18 x 150 x 5  |
| GP 2       | 40 x 3                  | 300x300          | 100x150           | 35 x 260 x 5  | 100x175           | 40 x 300 x 5  |
| GP 3       | 40 x 3                  | 300x300          | 50x100            | 21 x 174 x 5  | 100x100           | 27 x 174 x 5  |
| GP 4       | 40 x 3                  | 300x300          | 150x100           | 31 x 225 x 5  | 175x100           | 34 x 263 x 5  |
| TCP5       | 40 x 3                  | 450x300          | 0                 | "             | 100x100           | 26 x 245 x 5  |
| TCP6       | 40 x 3                  | 450x300          | 175x100           | 32 x 260 x 2  | 250x100           | 38 x 375 x 5  |

Notes: As for table under Figure 6.