"STUDY OF TURBULENT BOUNDARY LAYER BEHAVIOUR"

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A B S T R A C T

The main objective of this paper is to study the behaviour of turbulent boundary layer regarding the surface roughness. The existing theoretical and empirical methods as well as various experimental results have been reviewed and studied. A new simplified methodology is suggested to take into account the different behaviour parameters of boundary layer. The effect of such parameter and velocity profiles are well defined and given in the form of design charts.

Moreover, using these design charts a proposed numerical procedure is introduced to solve the momentum equation of turbulent flow over rough surface with the help of isocline method.

NOMENCLATURE

c_∞	free stream velocity,	m/s
$\bar{c}$	velocity at the outer edge of the boundary layer,	m/s
c_τ	friction velocity,	
c_f	local skin friction coefficient,	
c_x	velocity of the fluid inside the boundary layer in x-direction,	m/s
c_y	velocity component of the fluid inside the boundary layer in y-direction,	m/s
H_{12}	boundary layer form parameter	
I	boundary layer shape parameter	
K_s	roughness height,	m
P	velocity profile parameter	
p_∞	free stream static pressure,	N/m^2
$Re_{\delta^{**}}$	Reynolds number based on the momentum thickness	
$w(y/\delta)$	wake function,	
δ	boundary layer thickness,	m
δ^{*}	displacement thickness of the boundary layer,	m
δ^{**}	momentum thickness of the boundary layer,	N/m^2
τ	shear stress in the boundary layer,	N/m^2

τ_w	wall shear stress,	N/m^2
ν	kinematic viscosity of fluid,	m^2/s
Λ	Euler number	
π	pressure gradient parameter	
ρ	density of fluid,	kg/m^3

1- INTRODUCTION

This is a study of the behaviour of the turbulent boundary layer of incompressible fluids, regarding the surface roughness. Such study has been done in [1]for smooth surface In general the calculation of turbulent boundary layer is based on approximate methods. However, an exact solution of the Prandtl boundary layer equations is available only for special relations for $\bar{c}$ (x). The basis of such calculations depends upon the momentum equation of boundary layer. This equation has, in general case, two variable coefficients, namely, the form parameter H_{12} and the dimensionless wall shear stress $\tau_w/\rho\bar{c}^2$. These coefficients are dependent functions of Reynolds number $Re_{\delta^{**}}$ as well as Euler number Λ.

Many methods for calculating turbulent boundary layer regarding pressure gradient were introduced, checked and modified by several authors among them [2], [3], and [4]. In recent years, it is still required, for technical applications, to know more information about these variable coefficients for rough surface.

In this paper the behaviour of turbulent boundary layer for rough surface is theoritically studied and the parameters affecting this behaviour have been investigated.

2- BASIC ANALYTICAL RELATIONS

For fully turbulent boundary layer, the surface roughness has the major effect on the velocity profile. It should be noticed that, in this study Coles [5]neglected the effect of surface roughness on the velocity profile. In this case, it is suggested to expand Cole's definitions to take into account the surface roughness in the same manner as it has been considered by Rotta [6]. Accordingly, the proposed expression for the velocity profile will be:

$$\frac{c_x}{c} = \left(\frac{1}{\varkappa} \ln \frac{c_{\tau} . y}{\nu} + C \right) + \frac{P(x)}{\varkappa} w(y/\delta) + C_1\left(\frac{c_{\tau} . k_s}{\nu}\right) \quad ..(1)$$

It is clear from equation (1) that this representation of the velocity profile consists of the law of the wall, the law of the wake and the roughness effect. These are given by the first, the second and the third terms in the right hand-side of Eq.(1), respectively.

The thickness of the boundary layer is defined by:

$$c_x = \bar{c} \quad \text{for} \quad y = \delta ,$$

so that equation (1) can be rewritten in the form of the defect law as

$$\frac{\bar{c} - c_x}{c_\tau} = -\frac{1}{\varkappa}\ln(y/\delta) + \frac{P}{\varkappa}\left[2 - w(y/\delta)\right] - C_1\left(\frac{c_\tau . k_s}{\nu}\right) \quad ..... (2)$$

Rearranging equation (2), dividing by $\bar{c}$ and solving for $c_x/\bar{c}$, one obtains,

$$\frac{c_x}{\bar{c}} = 1 + \sqrt{\frac{\tau_w}{\rho\bar{c}^2}}\left\{\left[\frac{1}{\varkappa}\ln(y/\delta) - \frac{2P}{\varkappa}\right] + \frac{P}{\varkappa}w(y/\delta) + C_1\left(\frac{c_\tau . k_s}{\nu}\right)\right\} \quad ..... (3)$$

equation (3) is a modified form of the velocity distribution for turbulent flow regarding the effect of surface roughness. Also, it is one of the basic concepts for solving and estimating the different parameters affecting the behaviour of the turbulent flow.

Moreover, equation (1) represents the basis for the determination of the dimensionless wall shear stress $(\tau_w/\rho\bar{c}^2)$. At the outer edge of the boundary layer (i.e., $y = \delta$) $c_x = \bar{c}$, $w = 2$ and $C_1(c_\tau . k_s/\nu) \longrightarrow C$. The constant C in Eq.(1) was found experimentally [5] to have the value of 5.1. Therefore, Eq.(1) becomes:

$$\frac{\bar{c}}{c_\tau} = \frac{1}{\varkappa}\ln\left(\frac{c_\tau . \delta}{\nu}\right) + 5.1 + \frac{2P}{\varkappa} . \quad ..... (4)$$

The term $(c_\tau . \delta/\nu)$ which is a type of Reynolds number, will be expanded to have the following form:

$$\frac{c_\tau . \delta}{\nu} = Re_{\delta^{**}} . \frac{\sqrt{\tau_w/\bar{c}^2\rho}}{\delta^{**}/\delta}$$

The last obtained form of Reynolds number is substituted in equation (4) to give:

$$\frac{1}{\sqrt{\tau_w/\rho\bar{c}^2}} = \frac{1}{\varkappa}\ln Re_{\delta^{**}}\frac{\sqrt{\tau_w/\rho\bar{c}^2}}{\delta^{**}/\delta} + 5.1 + \frac{2P}{\varkappa} \quad (5)$$

This derived equation (5) is used in the computer programme of the present work to give the value of the dimensionless wall shear stress $\tau_w/\rho\bar{c}^2$.

3- EFFECT OF ROUGHNESS

The distribution of surface roughness and roughness height have a major effect in the development of the turbulent boundary layer. This effect is defined by the function $C_1(c_\tau . k_s/\nu)$ in Eq.(3). Therefore, the local skin friction coefficient c_f can conveniently be expressed as :

$$c_f = c_f(Re_{\delta^{**}},\ H_{12},\ k_s/\delta^{**}) \quad \ldots\ldots (6)$$

For small rouganess objects the value of C_1 is constant and is independent of the ratio k_s/δ^{**}. The surface can be regarded as smooth. With increasing heights of roughness, the ratio k_s/δ^{**} gains more and more influence relative to the Reynolds number $Re_{\delta^{**}}$, untill finally the skin friction coefficient becomes a function of k_s/δ^{**} and H_{12} only. In other words the dimensionless wall shear stress can be expressed as:

$$\frac{\tau_w}{\rho \bar{c}^2} = f_1(k_s/\delta^{**};\ H_{12}) \quad \ldots\ldots (7)$$

Also, the dimensionless wall shear stress $\tau_w/\rho\bar{c}^2$ can be expressed as a function of k_s/δ^{**}, and Euler number Λ, in the form ,

$$\frac{\tau_w}{\rho \bar{c}^2} = f_2(k_s/\delta^{**},\ \Lambda) \quad \ldots\ldots (8)$$

For the numerical solution of the velocity profile, it is necessary to know more information about the function $C_1(c_\tau k_s/\nu)$. This is clearly demonstrated by the exprimental Nikuradse's [7] sand grain roughness as follows:

A) For values of $c_\tau k_s/\nu < 5$,

$$C_1\left(\frac{c_\tau k_s}{\nu}\right) = 5.5 + 2.5\ \ln\left(\frac{c_\tau k_s}{\nu}\right) , \quad \ldots\ldots (9)$$

like completely smooth pipe. This is consistent with the idea that as long as the roughness is well immersed in the laminar sublayer it does not shed eddies and therefore can have no effect on the flow.

B) For values of $c_\tau k_s/\nu > 70$,

$$C_1\left(\frac{c_\tau k_s}{\nu}\right) = \text{constant} \approx 8.5 \quad \ldots\ldots (10)$$

In this case the flow and surface friction are independent of Reynolds number. The flow is referred to fully developed roughness flow.

C) The intermediate region is one in which the viscous friction and roughness form drag contributions to the surface drag are both significant and change in dominance from one to the other takes place as $c_\tau k_s/\nu$ increases from 5 to 70. So the representation of the transient regime between hydraulically smooth and completely rough surfac is more complex.

4- CHARACTERISTIC PARAMETERS OF THE BOUNDARY LAYER

4.1- FORM PARAMETER H_{12}

The displacement thickness, as it is well known, isdefined indimensionless form by:

$$\frac{\delta^*}{\delta} = \int_0^1 \left(1 - \frac{c_x}{\bar{c}}\right) d(y/\delta) \qquad \ldots\ldots (11)$$

From equation (3), substituting in equation (11) and integrating by parts, a convenient relation between the dimensionless displacement thickness and velocity profile parameter P, roughness effect and dimensionless wall shear stress $\tau_w/\rho\bar{c}^2$, is obtained as:

$$\frac{\delta^*}{\delta} = \frac{1}{\varkappa}\sqrt{\frac{\tau_w}{\rho\bar{c}^2}}\left[1 - P - \varkappa C_1 R_s\right] \qquad \ldots\ldots (12)$$

Also, the momentum thickness can be illustrated in the following shape:

$$\frac{\delta^{**}}{\delta} = \frac{1}{\varkappa}\sqrt{\frac{\tau_w}{\rho\bar{c}^2}}\,(1 + P - C_1 R_s) - \frac{2}{\varkappa^2}\cdot\frac{\tau_w}{\rho\bar{c}^2}\left[1 + 1.6P + 0.761P^2 + \varkappa C_1 R_s(2 + C_1 R_s - P)\right] \qquad \ldots\ldots (13),$$

where: $R_s = \frac{c_\tau k_s}{\nu}$

To have the conventional form parameter as a function of the velocity profile parameter P, dividing Eq.(12) by (13) gives

$$H_{12} = \frac{\delta^*/\delta}{\delta^{**}/\delta}$$

$$= \frac{1}{1 - \frac{2}{\varkappa}\sqrt{\frac{\tau_w}{\rho\bar{c}^2}}\left(\frac{1+1.6P+0.761P^2+\varkappa C_1 R_s(2+\varkappa C_1 R_s - P)}{1 + P - \varkappa C_1 R_s}\right)} \qquad \ldots\ldots (14)$$

This form of equation (14) is used in FORTRAN IV computer programme.

4.2- EULER NUMBER

The Euler number Λ is one of the most important boundary layer parameters. This importance can be seen from the different representation of the results obtained and given in the form of design charts. The definition of this parameter is given by:

$$\Lambda = -\frac{1}{\bar{c}}\frac{d\bar{c}}{dx}\,\delta^{**} \qquad \ldots\ldots (15)$$

4.3- THE SLOPE OF THE MOMENTUM THICKNESS

VON Kármáns momentum integral equation is performed to have the following form:

$$\frac{d\delta^{**}}{dx} = (H_{12} + 2)\,\Lambda + \frac{\tau_w}{\rho\bar{c}^2} \qquad \text{..... (16)}$$

Equation (17) is the slope of the momentum thiskness as a function of the form parameter H_{12}, Eulor number Λ , and the dimensionless wall shear stress.

5- REPRESENTATION AND DISCUSSION OF RESULTS

Based on the results given in figures from (1) to (4) the controlling parameters, which affect the behaviour of the turbulent boundary layer over a rough surface, could be found. In order to find the parameters, information to determine Reynolds number $Re_{\delta^{**}}$, Euler number Λ and the ratio k_s/δ^{**} have to be given. Therefore, the momentum thickness δ^{**}, The velocity gradient $d\bar{c}/dx$ and the height of surface roughness k_s should be known.

By the aid of the above figures, the form parameter H_{12} and the dimensionless wall shear stress $\tau_w/\rho\bar{c}^2$ may then be expressed as follows:

$$\frac{\tau_w}{\rho\bar{c}^2} = f_3(\Lambda\ ,\ Re_{\delta^{**}}\ ,\ k_s/\delta^{**}) \qquad \text{..... (17)}$$

$$H_{12} = f_4(\Lambda\ ,\ Re_{\delta^{**}}\ ,\ k_s/\delta^{**}) \qquad \text{..... (18)}$$

By increasing the heights of roughness, the ratio k_s/δ^{**} has more influence on $\tau_w/\rho\bar{c}^2$ and H_{12} than $Re_{\delta^{**}}$; and by increasing these heights of roughness further more, the local skin friction coefficient $\tau_w/\rho\bar{c}^2$ and the form parameter H_{12} become only functions of Euler number Λ and the ratio k_s/δ^{**} as given by equation (8).

Fig.(1) shows the relation between the local skin friction coefficient and Eulor number Λ for Reynolds number $Re_{\delta^{**}} = 3.10^2,\ 5.10^3,\ 10^4,\ 5.10^4,\ 10^5,\ 5.10^5$ and 10^6.

For boundary layers at the same Euler number, the local skin friction coefficient increases as the Reynolds number decreases. Also, as can be seen from the figure, for the low values of Euler number, there is small variation of the local skin friction coefficient versus large variation in Euler No.

Fig.(2) represents the variation of the local skin friction coefficient with Euler number for different values of the ratio k_s/δ^{**}. For the same value of the ratio k_s/δ^{**} the slope of the curve $dc_f/d\Lambda$ is too small. This means that, there is a variation in the local skin friction coefficient, corresponding to a large variation of Euler number. By increasing the ratio k_s/δ^{**}, the slope $dc_f/d\Lambda$ increases, i.e., a large variation in the local skin friction coefficient, corresponding to small variations of Euler number occurs.

The form parameter is plotted in Fig.(3): it represents the form parameter H_{12} versus Euler number with k_s/δ^{**} as parameter. The diagram consists of a family of parallel curves and shows that for the same value of the ratio k_s/δ^{**}, the form parameter H_{12} increases as the Euler number increases. This means that, by increasing Euler number, the dimensionless displacement thickness δ^{*}/δ increases while the dimensionless momentum thickness δ^{**}/δ decreases.

In Fig.(4) the relation between the slope of the momentum thickness $d\delta^{**}/dx$ and Euler number Λ for different values of $Re_{\delta^{**}}$ is represented. For constant value of Λ, the slope $d\delta^{**}/dx$ increases as $Re_{\delta^{**}}$ decreases. For low values of Euler number ($\Lambda < 3.10^{-3}$), the slope of each curv is positive and is approximately constant. This means that, the increase of the slope of the momentum thickness $d\delta^{**}/dx$ causes an increase of Euler number Λ.

6- CONCLUSIONS

The conclusions obtained from the present investigation will now be summarized. In the present work the proposed semi-empirical method for two-dimensional turbulent boundary layer takes into account the wall roughness. The results of computations are classified and organized according to Euler number Λ and the roughness parameter k_s/δ^{**}.

The result of computations have showed that the wall roughness k_s/δ^{**} has a greater effect than the boundary layer parameters H_{12} and $\tau_w/\rho\bar{c}^2$. As a result the total resistance increases. This is due to the fact that the boundary layer profile is strongly affected by the development of roughness.

The slope of the momentum thickness $d\delta^{**}/dx$ increases with the increase of Euler number Λ and decrease with the increase of Reynolds number $Re_{\delta^{**}}$.

It should be noted that, as a result of the present work, the momentum integral equation can be simply solved with the aid of the deduced diagrammes using the isocline method.

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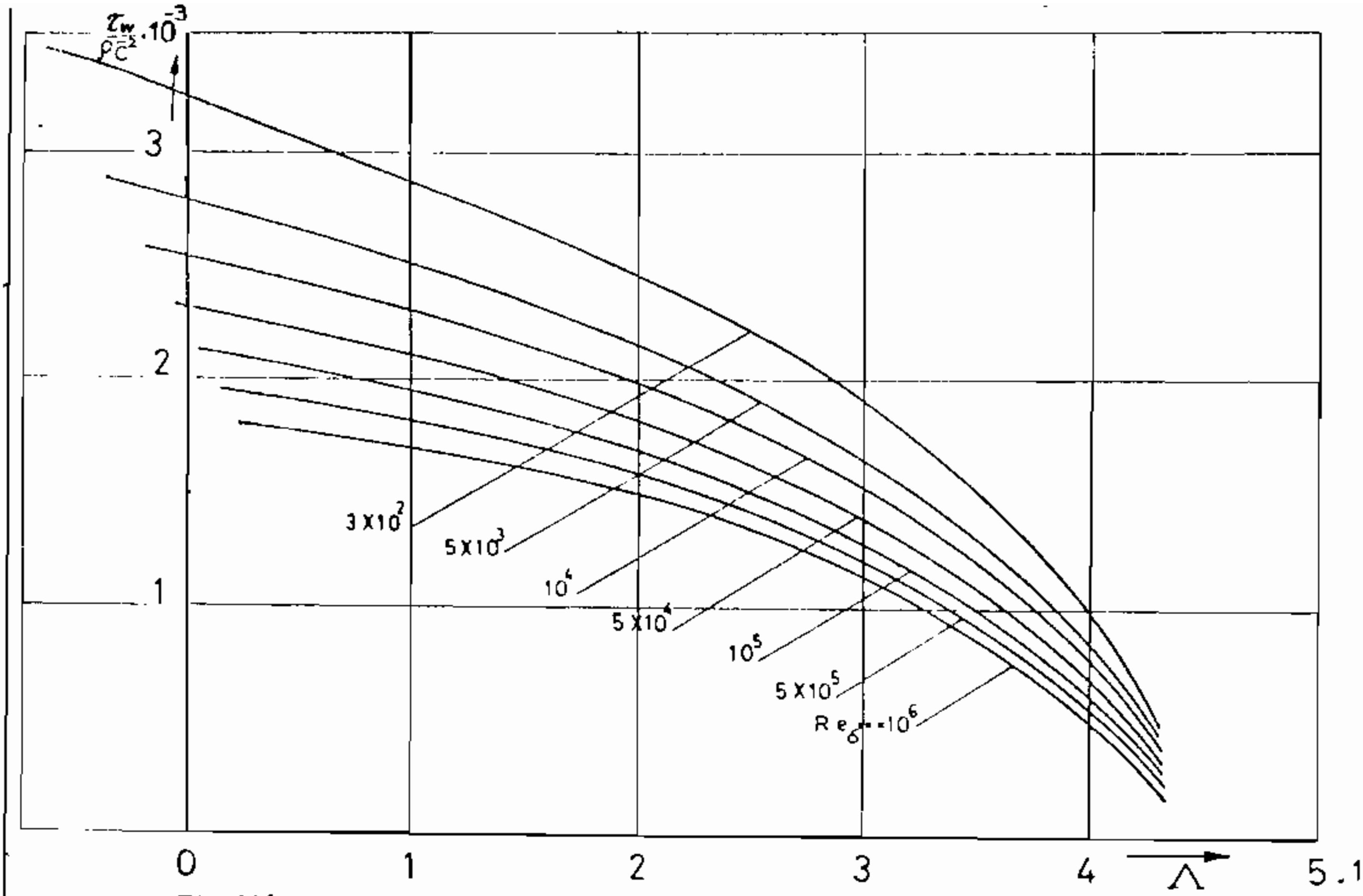

Fig.(1) Local Skin Friction Coefficient Versus Euler Number Λ With Reynolds Number $Re_{\delta^{**}}$ As Parameter

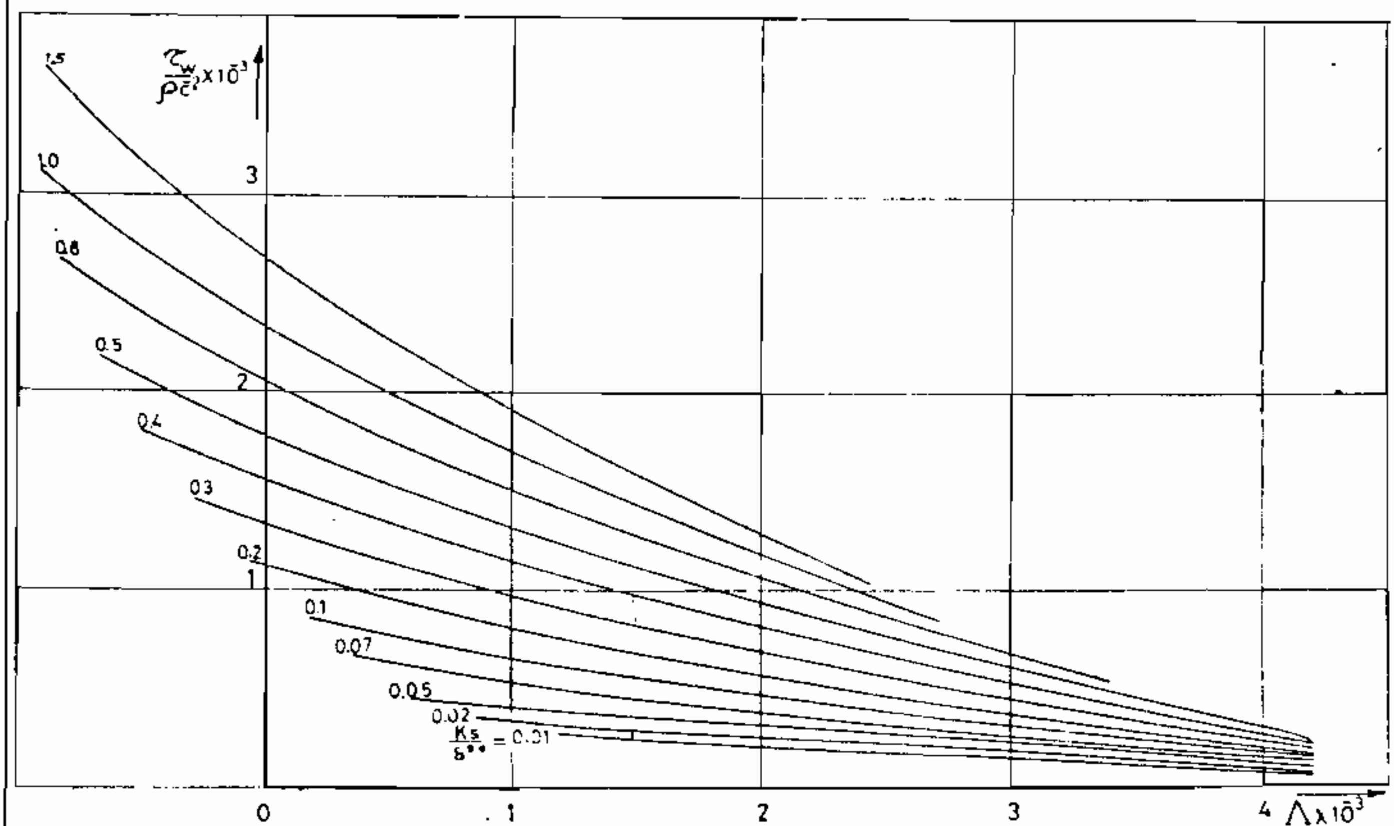

Fig.(2) Local Skin Friction Coefficient As Function of Euler Number With Ks/δ^{**} As Parameter

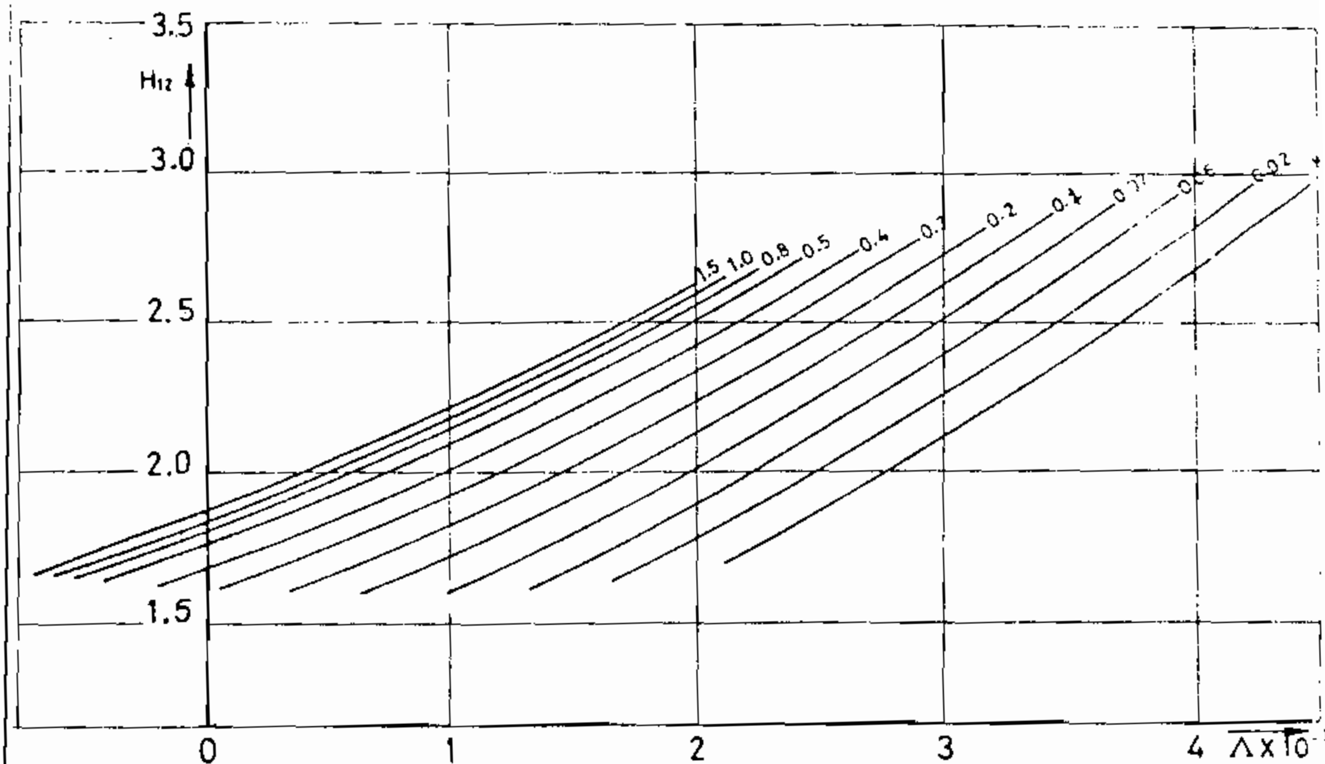

Fig.(3) Form Parameter H12 Versus Euler Number Λ with The Ratio Ks/δ** As Parameter

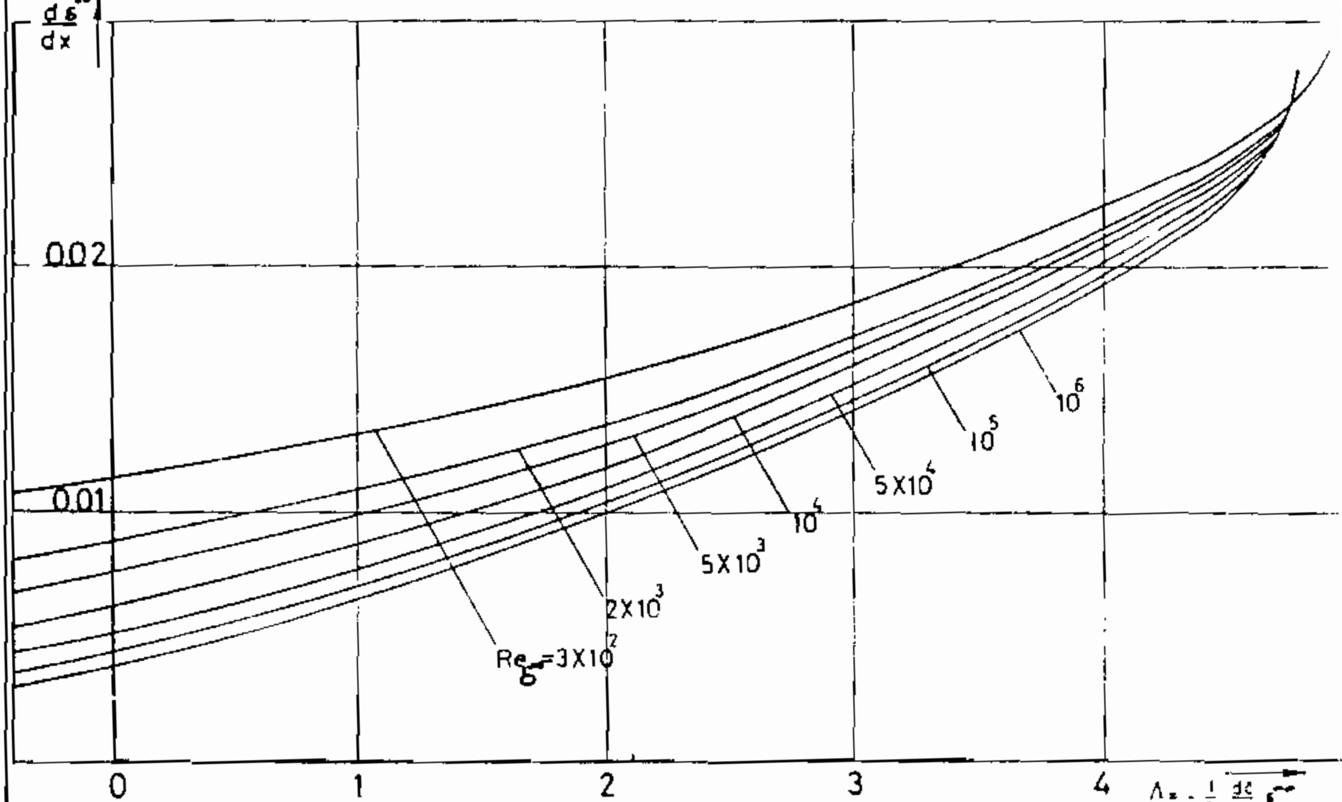

Fig.(4) Slape Of The Momentum Thickness As Function Of Euler Number For Different Values Of Reynolds Number $Re_{\delta^{**}}$