POLE ASSIGNMENT SCHUR METHOD FOR DESIGNING WIDE-RANGE CONTROL SYSTEMS

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I INTRODUCTION

Linear optimal control theory when used in the designing of multi variable control systems often encounters difficulties in practice [1,2]. The main difficulty is the large variation of the plants parameters, resulting from the dynamic operation of such plants. Under such a variation the dynamic stability of the system is not ensured.

Recently, methods have been suggested to overcome this difficulty through the introduction of feedback controllers suitable for dynamic operation of multi variable control systems. Elmetwally [3] introduces a suboptimal control system with low sensitivity to plants parameters by incorborating sensitivity consideration with respect to such parameters into the design of the optimal controller. Outhred [4] proposes an optimal controller with an additional correcting signal used to compensate for the small variations in the plants parameters. El-Desouky [5] has designed a self - caliberating optimal controller by which the gain settings of the controller are automatically adjusted to follow dynamic system change. However, these techniques suffer from the high computational effort required for solution and the complexity in system structure.

This paper presents a procedure and algorithm for selecting a control law that takes into account variation of plants parameters. This algorithm is based on pole - shifting technique and uses the Schur Form of the system state matrices, which represent the system under different operating conditions, for designing a feedback control law with low sensitivity to system parameter variations and disturbance input. The technique shifts only the bad eigenvalues of the system and hence the resulting gains of the control system are minimized.

II POLE-SHIFTING TECHNIQUE FOR WIDE-RANGE CONTROLLERS

The linear mathematical model representing the dynamic systems under different operating conditions may take the form :

$$\dot{X}_i = A_i X_i + B_i U_i \qquad i = 1,2 \ldots\ldots m \qquad (1)$$

where,

$A \in R^{nxn}$ is a constant matrix containing parameters depending on operating conditions,

$B \in R^{nxr}$ is a constant matrix,

$U \in R^{r}$ is a control vector,

$X \in R^{n}$ is a state vector, and

m representing the assumed number of operating conditions.

The aim of this paper is to determine the constant feedback matrix F in the control laws

$$U = FX_i \qquad (2)$$

such that the following closed-loop systems are achieved :

$$T(A_i + B_i F) = D \qquad i = 1,2,----m \qquad (3)$$

where,

D is the desired eigenvalues, and

T (A + BF) is the spectrum (the set of n eigenvalues).

A preliminary step of the proposed algorithm for computing F is a reduction of the set A_i into the Real Schur Form (RSF):

$$A_i = \begin{bmatrix} A_{1i} & A_{3i} \\ & A_{2i} \end{bmatrix} \qquad (4)$$

while the set B_i is devided into :

$$B_i = \begin{bmatrix} B_{1i} \\ B_{2i} \end{bmatrix} \qquad (5)$$

If the set of pair A_i and B_i is controlable, then (A_{2i}, B_{2i}) is controlable. Assuming a feedback matrix F in the form of $F = [0 \;\; F2]$, we can only modify the eigenvalues corresponding to A_{2i} in the resulting set of closed-loop state matrices :

$$A_i + B_i F = \begin{vmatrix} A_{1i} & A_{3i} + B_{1i} F_2 \\ & A_{2i} + B_{2i} F_2 \end{vmatrix} = D \quad (6)$$

As a result we can shift one or two pole of the last diagonal block of the RSF of the set of matrices A_i. Using the average spectrums of the linear set of models given by equation(6) the elements of the feedback matrix F is computed in the following way.

1) Choose any initial value for F.
2) Reduce and arrange the set of closed-loop state matrix A_{ci} = $A_i + B_i F$ to RSF, using orthogonal transformation U as follows:

$$A_{ci} \longrightarrow U_i A_{ci} U_i^t = \begin{vmatrix} A_{1i} & A_{3i} + B_{1i} F_2 \\ 0 & A_{2i} + B_{2i} F_2 \end{vmatrix}$$

Where :

$$A_1 \in R^{q \times q}, \quad A_2 + B_2 F_2 \in R^{(n-q)\times(n-q)}, \quad T(A_1) \subset D_g,$$

$T(A_2 + B_2 F_2) \subset D_b$, and D_g and D_b specify good and bad regions of D respectively.

Notice that the eigenvalue is considered good when it is good in all m systems otherwise it is a bad one.

3) Set i = q + 1
4) If i > n stop
5) Set H_i equals to the last block in A_{ci} of order p (p = 1 or 2) and set G_i equals to the last p rows of B_i
6) Use G_i to compute $K_i \in R^{r \times p}$ i = 1,2,3 ....m, using procedure given in section III to shift the set of P poles form D.
7) Compute $A = A_{ci} + B_i[0 \; , \; K_i]$ and $F = F + \frac{1}{m} \sum_{i=1}^{m} [\, 0 \; , \; K_i \,] \, U_i$
8) Move the last blocks of A_{ci} in positions (i,i) accumulating the transformations in U and compute $B_i \longrightarrow U_i B_i$
9) Set i = i + p and go to step 4.

III PROCEDURE FOR POLE-ASSIGNMENT

Consider the controllable pair (C,G) where $C \in R^{pxp}$, $G \in R^{pxm}$ and p = 1 or 2. Let Γ_p be a symmetric set of p complex numbers and r = rank (G). The feedback matrix $K \in R^{mxp}$ which assigns Γ_p may be calculated as follows.

1- Compute $G = Q\,(\hat{G}\;\; 0)\,V'$ where $\hat{G} \in R^{pxr}$ is upper right triangular, and $Q \in R^{pxp}$, $V \in R^{mxm}$ are orthogonal matrices.

2- Compute by similarity $\hat{C} = Q'\,C\,Q$

3- If r = p, compute $\hat{K} = \hat{G}^{-1}(J - \hat{C})$ where $J \in R^{pxp}$, $\Gamma(J) = \Gamma_p$ otherwise compute $\hat{K} = [\hat{k}_1, \hat{k}_2]$

where, $\hat{k}_1 = (b_1 + b_2 - a_{11} - a_{22}) / z$,

$\hat{k}_2 = (a_{22} / a_{21})\,\hat{k}_1 + (a_{11} a_{22} - a_{12} a_{21} - b_1 b_2)/a_{21} z$, and

$$\hat{C} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad \hat{G} = \begin{bmatrix} z \\ 0 \end{bmatrix}$$

4- Compute $K = V \begin{bmatrix} \hat{k} \\ 0 \end{bmatrix} Q'$

The singular value decomposition of G can be used to compute the factorization at step 1). However, in our case (P=1 or 2), it is more convenient to use the following particular factorization. The orthogonal matrix Q for P = 1 is simply Q = 1. If m = 1, then V=1. For P = 2, U can be chosen to be a rotation of the form :

$$Q = \begin{vmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{vmatrix}$$

The matrix V can be determined in a factored form of two Hermitian reflectors each having the form $I + aww'$ where $w \in R^m$ and $a = 2/(w'w)$. Standard techniques to compute Q and V are described in [6].

IV NUMERICAL EXAMPLE

A single machine infinite-bus unit is considered. Both exciter and govrnor controls are presented. The linearized state equations of the system take the form :

$$\begin{vmatrix} \dot{E} \\ \dot{\delta} \\ \dot{W} \\ \dot{P} \end{vmatrix} = \begin{vmatrix} \frac{1}{T_{d0}\, b_2} & 0 & \frac{-C2}{b2} & 0 \\ & & & \\ -\frac{b_1}{M} & -\frac{C_1}{M} & -\frac{D}{M} & \frac{1}{M} \\ 0 & 0 & 0 & \frac{-1}{T_g} \end{vmatrix} \begin{vmatrix} E \\ \delta \\ W \\ P \end{vmatrix} + \begin{vmatrix} \frac{1}{T_{d0}\, b_2} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \frac{1}{T_g} \end{vmatrix} \begin{vmatrix} U_1 \\ U_2 \end{vmatrix}$$

Where

$$c_1 = \frac{\partial Pe}{\partial \delta} = \frac{EV}{(x_d + x_1)} \cos \delta$$

$$b_1 = \frac{\partial Pe}{\partial E} = \frac{V}{x_d + x_1} \sin \delta$$

$$c_2 = \frac{\partial E}{\partial \delta} = - \frac{V(x_d - x_d')}{(x_d + x_1)} \sin \delta$$

$$b_2 = \frac{\partial E'}{\partial E} = 1 - \frac{(x_d - x_d')}{(x_d + x_1)}$$

E machine terminal voltge,

δ rotor angle,

w rotor speed, and

p electrical power.

The system parameters are;

M = 0.02 pu, D= 0.02 pu, Xd=1.0 pu, X_d = 0.3 pu

T_g = 1.0 sec, Td = 5.0 sec, Xl = 0.6 pu see Ref.(2) for more details. The proposed technique is used to design the feedback parameters of the wide-range controller in the range of P+JQ = 1+J0.5 and 1-J0.5. The desiered eigenvalues are taken as -7+J8, 4.0+J0, and 2+J0. For five operating conditions(m=5:1+J0.5, 1+J0.25, 1+J0, 1-J0.25, 1-J0.5) five systems are simulateed and the feedback matrix F that stabilize all systems is found to be

$$F = \begin{vmatrix} -1.285 & 0.5642 & 3.245 & -8.842 \\ -1.416 & 0.7002 & -2.1102 & -14.213 \end{vmatrix}$$

A three phase fault is applied at the terminals of the machine cleared in 0.05 second. The resultant system responses under the operating cases (1 + J 0.5, 1 + J 0, 1 - J 0.5) with the proposed controller in operation, are shown in figure (1). This figure shows satisfactory system responses under the assumed operating condition.

IIV CONCLUSION

A simplified control system that is suitable for controlling over wide range of system operating conditions is presented. The gains of the control law is calculated such that a pre-assigned set of eigenvalues are achieved. The method of selection of feedback coefficients is based upon orthogonal transformations which are most economical in computational requirements. The bad eigenvalues of the system is only modified and in this way the resulting feedback coefficients are minimized.

REFERENCES:

(1) Habibullah, B., Yec-nan Yu, "Physically Realizable Wide Power Range Optimal Controllers for Power Systems", IEEE. Trans. PAS 93. PP. 1498 - 1506 (1974).

(2) Kumar,A.B, et al"An Optimal Control Law Assignment For Improved Dynamic Stability of Power & Systems", IEEE, Trans. Pas 101 PP. 1570 - 1576 (1982).

(3) Elmetwally,M.M., et.al., "Extension of Stable Operating Regions of Synchronous Machine Using Low-Sensitivity Excitation Control", Proc. IEE. 121, PP. 1141 - 1145 (1974).

(4) Outhred. H.R. and Evans, F.J, " A Model Reference Adaptive Controller For Turbo - Alternators in Large System " 1972, 4th PSCC, Grenoble France.

(5) Eldesouky, A. I., Elkonaiy, E., " Continuous Acting Self-Caliberating Optimal Controller for Synchronous Machines Dynamic Operation "International Amse, Conference,1982 VOL.(2).

(6) Varga, A., "A Schur Method Pole Assignment" IEEE Trans vol. AC 26 PP 517-519 , 1981.

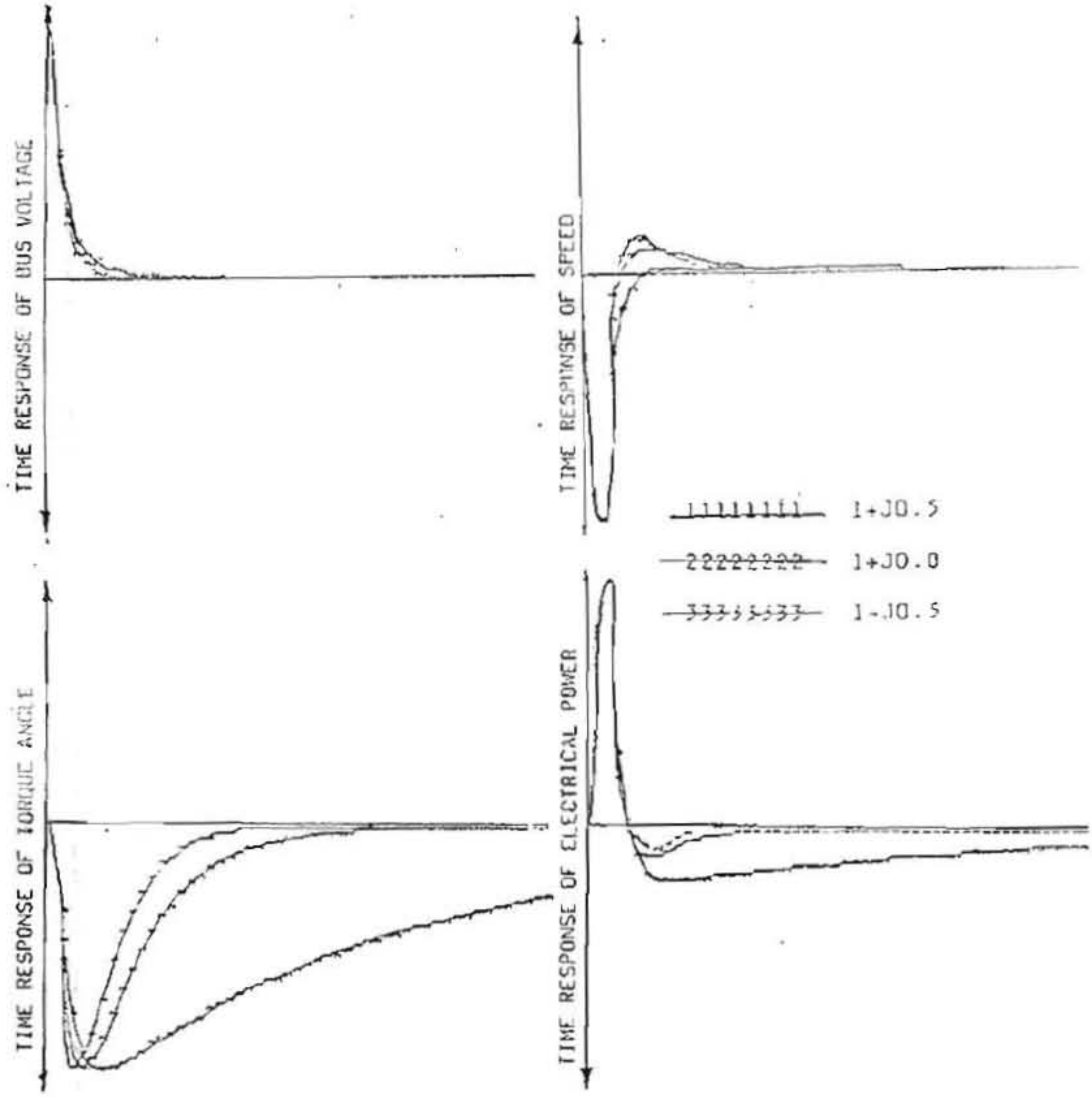
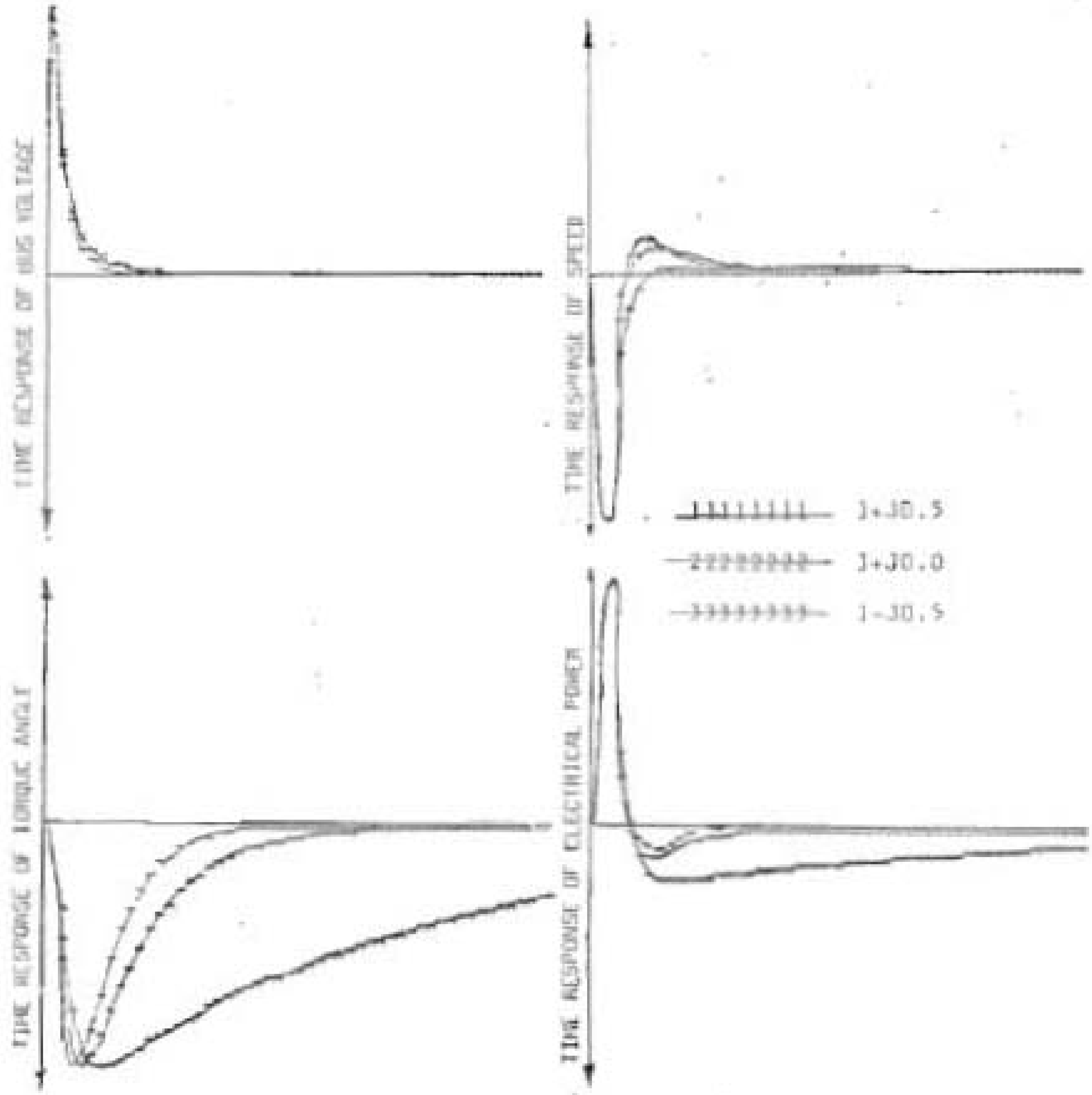


FIGURE (1): TIME RESPONSE OF THE CONTROLLED SYSTEM.



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