THE OPTICAL ANALYSIS OF AN ASYMMETRICAL PIECE-WISE CONCENTRATOR

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خلاصه – يقدم البحث تحليلا ضوئيا لنوع من المجمعات الشمسيه ذات نسبة تركيز منخفضه ، ويتكون من قطع صغيره مختلفه من المرايا مرتبه بطريقه معينه لتعكس الاشعاع الشمسى بتوزيع محدد مسبقا على مستقبل مسطح . وهذا التوزيع قد اختير بحيث يكون منتظما على جزء معين من المستقبل . وقد استنتجت المعادلات التى تحدد ترتيب القطع العاكسه ، وأيضا تم عمل برنامج حاسب يمكن عن طريقه ايجاد توزيع شده التركيز على سطح المستقبل فى أى وقت من اليوم بطريقة تتبع الشعاع . وقد تم عمل جهاز ذات نسبة تركيز كليه ٣٫٦٤ حيث ثبتت بعض المقاومات الضوئيه على المستقبل لقياس شده الاشعاع فى نقط مختلفه على سطحه بقياس شده التيار المار بها من بطاريه صغيره وحسبت نسبة التركيز الموضعيه فى هذه النقط بمقارنتها بالقياسات المأخوذه من مقاومه ضوئيه أخرى موجهه الى الشمس ، وقد قورنت النتائج العمليه والنظريه .

ABSTRACT- In this work, the optical analysis of a piece-wise concentrator (PWC), of the low concentration class, is presented. This concentrator, can be easily fabricated from known unequal length mirror segments, arranged in a certain order to form its reflecting surface, which is capable of producing a predetermined flux distribution over a flat receiver surface. The construction equations of the transverse shape of the reflecting surface, which produce a uniformly illuminated portion on a flat receiver around the noon time, are derived from the optical geometry principles. Also, the hourly flux distribution over the receiver surface is numerically predicted by the ray tracing technique, with an especially made computer program. The theoretical results are compared with the experimental data collected from outdoor tests on a prototype concentrator, which has a maximum concentration ratio of 3.64, designed and built for this purpose. The flux distribution is measured in terms of the currents (from a small battery) through some photo resistances, located at different points on the receiver surface. The local concentration ratio at these points is calculated as the local flux divided by that measured by another photo resistance directed towards the sun. The experimental data are given in graphical form .

1. INTRODUCTION

Diurnal tracking systems used in the focusing concentrators are too expensive, especially in large installations. Therefore, the stationary nonimaging concentrators had been proposed and taken a considerable amount of interest in the last few years. These concentrators which accept radiation over a range of angles without diurnal tracking may be subjected to occasional tilt adjustment to improve its performance. The first of these is the compound parabolic concentrator CPC, with one side illuminated flat receiver, has been introduced and investigated by Winston (1). The reflector sides of the CPC are of different parabolic surfaces, and its maximum concentration ratio CR_m, is a function of the acceptance angle, A as given by,

$$CR_m = 1/\sin (A/2)$$

However, Winston and Hinterberger (2) have shown that the absorber of a two dimensional ideal concentrator need not to be flat and parallel to the aperture. They prooved that, radiation incident within the acceptance angle on an aperture of width l, can be concentrated onto any convex receiver of a cercumference = l sin (A/2). They also proposed some receiver shapes which have the advantage of eleminating back losses. Asymmetrical nonimaging cylinderical concentrators are also available. The reflector may be a combination of a part of a circle followed by a parabolic segment (3). The receiver in this type is illuminated from one side and has one edge in the common focus. On the other hand, the receiver in the semi-parabolic concentrator SPC (4) is a fin along the axial plane. The maximum concentration ratio of this type is given by,

$$CR_m = 2/ \tan (A/2)$$

The overheating effect at the focal line in SPC can be avoided by using the reflector defocusing walls, which increases the cost and number of reflectoins. The daily variations in the CR is also high compared to that of CPC.

In order to increase the acceptance angle and overcome the technical problems in obtaining the parabolic shape in fabricating the CPC, Jones and Anderson (5) have proposed the compound circular arc concentrator CCAC. In fact this is a truncated CPC with its reflector sides approximated to circular aluminized plastic films. Grillo (6) has studied a new model with two or more channels of reflecting walls, to decrease the height and maintain larger CR. However, this concentrator which requires a high quality reflecting material with a carful assembly, has an increased average number of reflections relative to that of CPC.

Another design trend has been started to define a transverse shape of the reflecting surface, which results in a uniform flux distribution over the receiver surface. Gupta et al (7) have derived an analytical expression for the transverse shape of the reflecting surface (instead of the parabola in the CPC design) so that the flux distribution on the receiver is uniform for certain angles of incidence. The illumination becomes increasing nonuniform as solar radiation deviates from the axial plane. The analysis involves some algebraic equations from which the reflector shape can be numerically predicted. However, during their experimental work on a prototype model, they have observed 12% variation in the flux distribution, which is measured in terms of the current through three photodiods placed on the receiver surface. They also concluded 50% and 33% decrease in the reflector height and CR respectively, compared to the corresponding values in the CPC. However, they reported a very low experimental data, compared to the predicted results. But the analysis can be considered as an important and promising design trend. Therefore, the work in this line should be continued with other designs such as asymmetrical concentrator , having mirror segments forming its reflecting surface, which is the object of this work.

2. CONSTRUCTION EQUATIONS

Figure 1 shows the transverse shape of the reflecting surface and receiver of the proposed piece-wise concentrator PWC, which consists of carfully arranged unqual flat mirror segments, starting from the receiver bottom. The x-y coordinates are chosen such that when the incident beam radiation is parallel to the y-axis, all the reflected flux is distributed uniformly over a pre-defined portion R_o on the receiver as shown in the figure. The following definitions are necessary to derive the construction equations of the PWC reflecting surface:

1. The optical axis (axial plane) has the direction of maximum CR. All the incident radiation parallel to the axial plane (y-axis) hit the upper portion of the receiver R_o.

2. The acceptance angle A is the angle between the incident and reflected beams at the end point of the last segment. Therefore, the value of A depends on the total number of segments, N and can be obtained from the optical geometry as,

$$A = 180 - 2 S_N \quad \ldots (1)$$

where S_N is the angle between the segment uumber N and the x-axis.

3. The maximum aperture area is that corresponding to the maximum concentration ratio CR_m. Therefore,

$$CR_m = (r_N \sin A) / R \quad \ldots (2)$$

where R is the receiver width and r_N is the distance from the receiver top point to the last segment end point.

Now, the width of the first segment can be calculated from,

$$L_1 = R \sin (2 S_1 - W) / \cos S_1 \quad \ldots (3)$$

and the distance r_1 is given by,

$$r_1 = R \cos (S_1 - W) / \cos S_1 \quad \ldots (4)$$

In general, the corresponding values for the segments number 2,3,...., N can be obtained from,

$$S_n = 1/2 \tan^{-1} \left[\frac{\sin 2 S_{n-1} - (R_o / r_{n-1}) \sin W}{\cos 2 S_{n-1} - (R_o / r_{n-1}) \cos W} \right] \quad \ldots (5)$$

$$r_n = r_{n-1} \cos (2 S_{n-1} - S_n) / \cos S_n \quad \ldots (6)$$

$$L_n = R_o \sin (2 S_n - W) / \cos S_n \quad \ldots (7)$$

by taking the value of n = 2,3,...., N respectively. Thus, in order to construct the reflector, the receiver parameters R, W and the image length at maximum concentration, R_o as well as the angle of the first segment, S_1 should be arbitarly decided. The geometrical condition ($S_1 > W/2$) must be considered in the choise of angle S_1.

3. OPTICAL ANALYSIS

If the concentrator is fixed with its length in the E-W direction, and tilted in such a way to keep its axial plane pointing to the sun at noon position, this situation is called α -orintation (4). The present analysis is constrained with this type of orintation, which yields maximum CR at noon and decreased values at off-noon times, compared to other orintation mechanisms. In this case, the optical axis makes an angle Z with vertical, given by,

$$Z = \phi - \delta \quad \ldots (8)$$

The instantanious value of CR can be easily obtained from the optical geometry at any off-noon position as,

$$CR = CR_m \sin(A - \sigma) \cos Z_p / \sin A \quad \ldots (9)$$

where σ is the angle between the solar noon position and projection of the sun ray on the N-S vertical plane at any time,
and Z_p is the projection of Zenith angle on the E-W vertical plane.

However, the angle σ variations affects the flux distribution in the receiver width direction, while the change in Z_p, affects it only in the length direction (i.e. causes the end losses). For simplicity, the latter effect is not considered in the theoretical analysis.

At noon position, all segments except the first one reflect beam radiation on the portion R_o. The flux on the remaining area is only due to the first segment and reflected diffuse radiation. At any off-noon position, the falling beam radiation makes an angle σ with the axial plane, and that reflected from segment number n will hit the receiver between two limits defined by the distances U_n and D_n measured from the top point as shown in Fig.2. From the optical geometry of the system, it can be shown that,

$$U_n = r_n \sin\sigma / \sin(2S_n + \sigma - W) \quad \ldots (10)$$

and

$$D_n = r_{n-1} \sin(2S_n - 2S_{n-1} + \sigma) / \sin(2S_n + \sigma - W) \quad \ldots (11)$$

If a part of (or all) the reflected beam misses the receiver lower edge (D_n or $U_n > R$), it will be re-reflected on any other segment m, again to the receiver. This is the case of double reflection, where the following must be calculated :

a- The segment number, m

b- The point of intersection between the segment m, and the reflected beam, and

c- The position of the re-reflected beam on the receiver (the new values of U_n and D_n for the re-reflected beam).

If E is the angle between the relfleted ray extension and the x-axis, the segment number m can be obtained by trial and error, based on the condition $S_{m+1} \gg 180\text{-}E > S_m$, as illustrated in Fig.3. Consequently, the ordinates of the point of intersection with this segment, (x_p, y_p) can be written as,

$$x_p = x_o - y_p \cot E$$

$$y_p = (x_o + y_m \cot S_m - x_m)(\sin E \sin S_m) / \sin(E + S_m) \quad \ldots (12)$$

where x_m and y_m are the ordinates of the segment number m top point,

$$x_o = -(U_n - R)\cos(E + W) / \sin E$$

and,

$$E = 3\pi/4 - 2S_n - \sigma$$

On the other hand, the position of re-reflected ray on the receiver can be defined by the distance B from the top point as given by,

$$B = R - y_p / \cos W - (x_p + y_p \tan W)\sin T / \cos(W + T) \ldots (13)$$

where,

$$T = \pi - 2S_m - E$$

However, the condition for double reflection should be checked after each procecess of ray tracing to calculate for triple or multi-reflection cases if exist.

The flux density at any point on the receiver surface F_b, resulting from

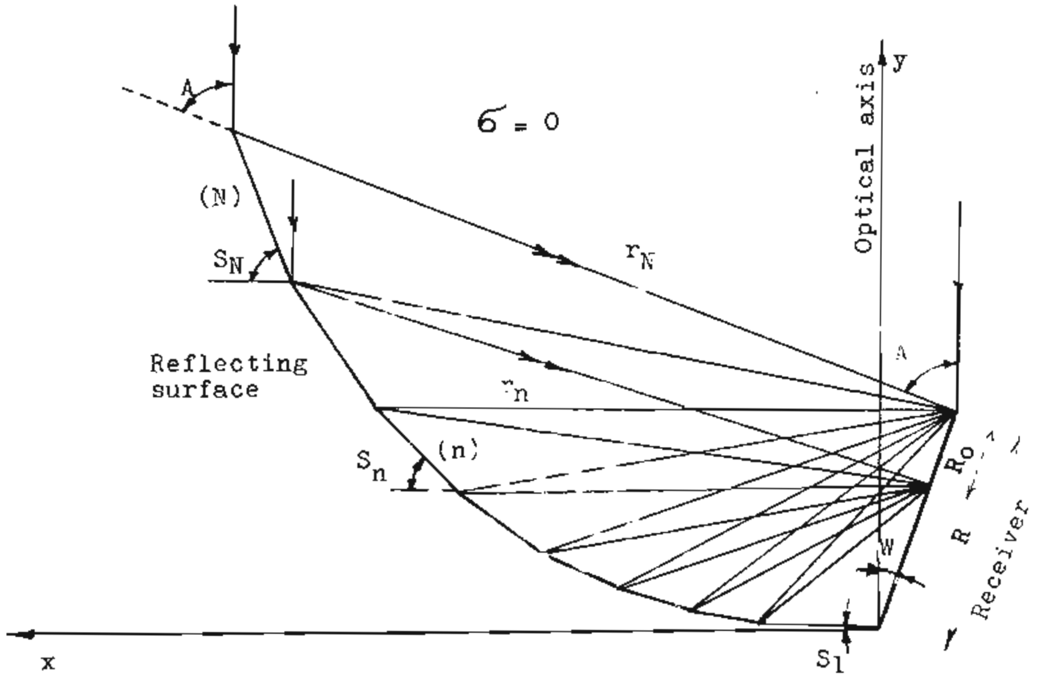

Fig. 1. Optical geometry of the PTC.

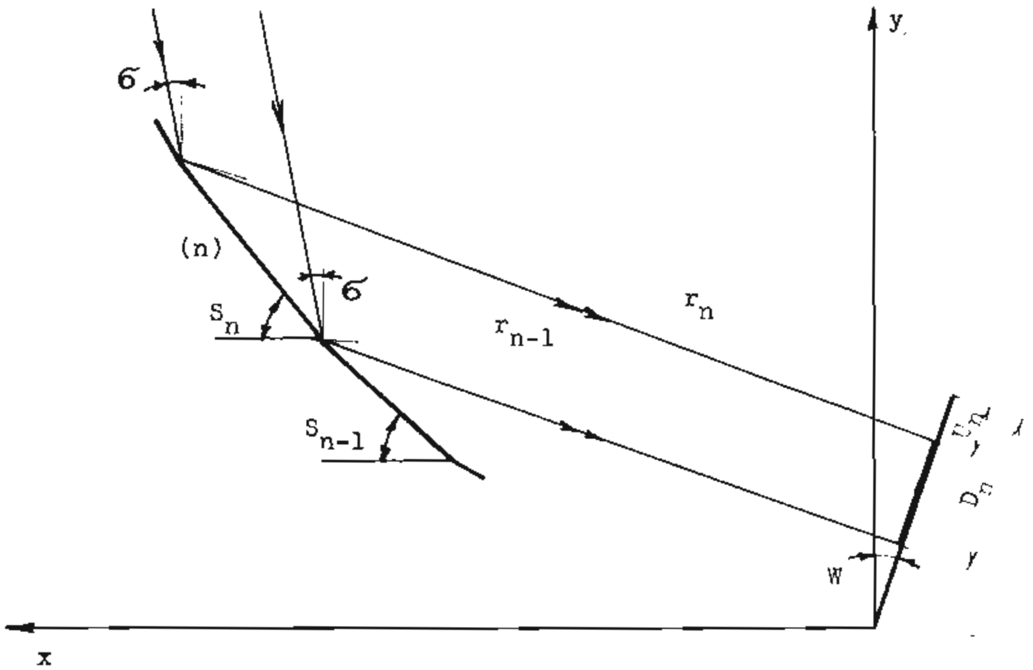

Fig. 2. Case of single reflection on the segment number (n).

the single, double or multi-reflections of beam radiation at any time, can be calculated from,

$$F_b \quad \sum_{n \quad 1}^{N} \quad I_{b,n} \; \rho^{j} \cos \theta_n \qquad \ldots (14)$$

where, $I_{b,n}$ is the normal beam radiation intensity,

ρ is the reflectivity of the mirror segments,

θ_n is the incident angle of the reflected beam from the segment n on the receiver at that point = T + W,

and j is the number of reflections to which the beam is subjected.

However, diffuse radiation intensity on horizontal plane, I_d is added to F_b after being multiplied by the correction factor, obtained from the latest reported infromation (8). Accordingly, the total flux can be calculated from,

$$F = F_b + I_d [(2 + \cos \beta) / 3 + \gamma (1 - \cos \beta) / 2] \qquad \ldots (15)$$

where β is the receiver tilt angle to the horizontal = 90 + Z - W,

and γ is the diffuse reflectance on the concentrator and surrounding space (taken as 0.8 in the analysis)

Local concentration ratio at any point on the receiver (in the width direction) is obtained from

$$cr = F / I_n \qquad \ldots (16)$$

where I_n is the normal total solar radiation intensity, which is given by,

$$I_n = I_{b,n} + I_d (2 + \cos Z) / 3 \qquad \ldots (17)$$

The procedure is repeated to get cr at different points yielding finally the flux distribution over the receiver in its dimensionless form, which is a more useful design tool. Also, calculations can be repeated, changing the time to obtain the hourly dimensionless flux distribution for any day of the year. This is carried out by the computer program. It is to be noted that the flux distribution in length direction depends mainly on the concentrator geometry and orintation.

4. EXPERIMENTAL SET UP AND PROCEDURE

An experimental set up has been designed and fabricated with a flat receiver of width R = 25 cm, and N = 19 mirror segments forming its reflecting surface, to verify the theoretical results. The receiver, which has a length of 94 cm makes an angle W = 22° with the optical axis. The first mirror segment is adjusted to make an angle S_1 = 12° with the x-axis. According to these data and the construction equations 3 to 7, the mirror segments are arranged as shown in Table 1. The segments, which are of 1 m length, are fixed to three wooden pieces, having the same transverse shape of the reflecting surface. The whole assembly is then supported on a wooden frame, which is capable of changing the concentrator orintation, as shown in Fig. 4.

The flux distribution is measured in terms of the current flowing through 10 photo resistances placed widthwise and lengthwise, on the receiver surface as shown in the figure. The local concentration ratio is calculated as the ratio between the flux at these points and that measured by another photo resistance tracking the sun. The normal beam radiation is measured by a pyrheliometer, while the horizontal total solar radiation is measured by another pyranometer. Diffuse radiation is calculated as the difference between the total and beam radiation, refered to the horizontal plane.

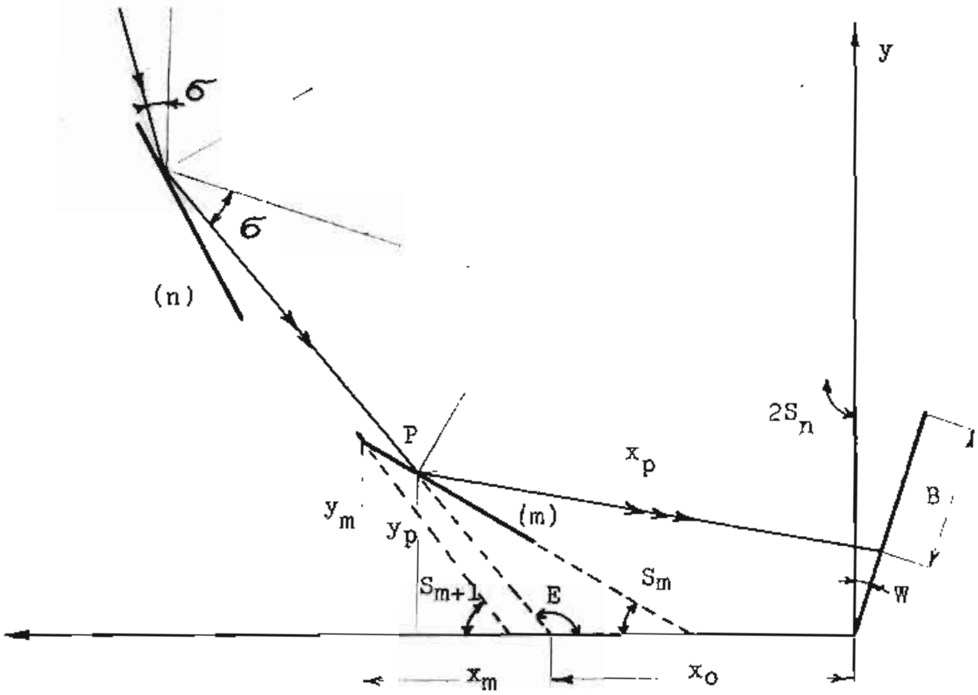

Fig.3. Case of double reflection on segments (n) and (m).

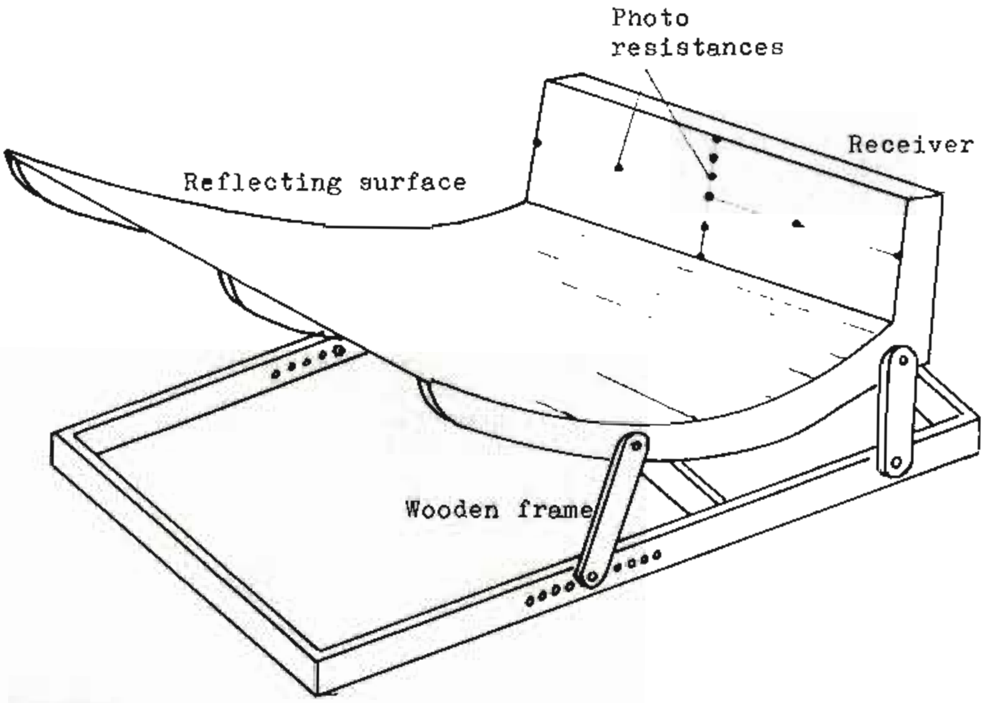

Fig.4. The experimental set up.

segment number	width L, cm	angle S°	segment number	width L, cm	angle S°
1	0.9	12	11	7.0	35.530
2	0.4	12.424	12	8.5	40.816
3	0.5	13.02	13	10.1	45.847
4	0.8	13.874	14	11.5	50.360
5	1.0	15.063	15	12.8	54.258
6	1.5	16.714	16	13.9	57.562
7	2.2	18.967	17	15.0	60.346
8	3.0	21.962	18	15.9	62.695
9	4.1	25.785	19	16.7	64.689
10	5.4	30.388			

Table 1. Specifications of the reflecting surface

The experiments are performed at Mansoura University, where the latitude angle ϕ = 31°, during the first week of April (declination angle δ = 3.5° as an average value). According to equation 8, the zenith angle Z = 27.5°, and the orintation of concentrator is fixed so that the receiver makes an angle Z - W = 5.5° with the verticial direction. The experimental data are recorded daily from 9 a.m to 4 p.m.

5. RESULTS AND DISCUSSION

The local concentration ratio cr, (calculated from the measured photo resistances current ratio) on the receiver surface in the width direction is shown in Fig. 5. At noon time, the distribution of cr is almost uniform over the upper third of the receiver, with an average value of about 2.5, and then gradually drops to about 0.8 over the remaining part. In the period from 11 a.m to 1 p.m, this distribution is seen to be unchanged, except a little shift of the peak downwards. Before and after this period, the peak value drops and is shifted downwards as the sun travels away from noon position. However, it is clear from this figure that the flux over the receiver lower half is always low during the day time. If the concentrator is properly orinted, the above distribution will not have a considerable seasonal change. Therefore, the receiver width should be reduced to the upper half, which in turn decreases the cost, specially in photovoltaic applications.

On the other hand, a considerable change in the local concentration ratio is observed in the receiver lengthwise direction at off-noon positions, due to the E-W motion of the sun. Fig. 6 shows the distribution at 10 a.m, 12 noon and 2 p.m, which gives an indication of the optical end losses, resulting from the finite length of the receiver.

The theoretical results corresponding to the given geometrical conditions are obtained. The measured solar radiation data shown in Fig. 7 are used in the prediction. The value of the mirror reflectivity is also measured (ρ = 0.81) to obtain accurate results. But a large difference is observed between the predicted and measured values. This is clear in Fig. 8, which showns a sample of measured data and the corresponding predicted results. The measurements have shown a maximum value of 2.7 for cr, corresponding to a predicted value of 6.65. However, Gupta et al (7) have observed an experimental value of about 2 compared to the corresponding predicted value of 11. This difference is probably due to the reflecting material defects and assembly errors.

6. CONCLUSIONS

The given theoretical model can be considered as a suitable design technique for the proposed asymmetrical piece-wise concentrator,which produces a pre-defined

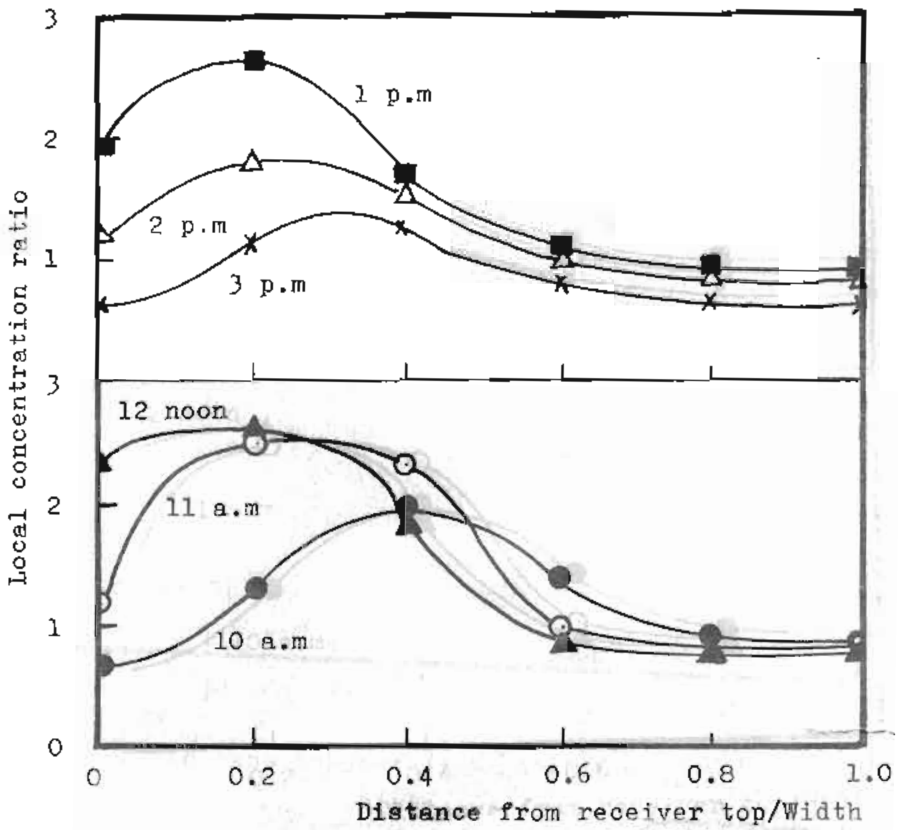

Fig. 5. Local concentration ratio in the receiver width direction at different hours.

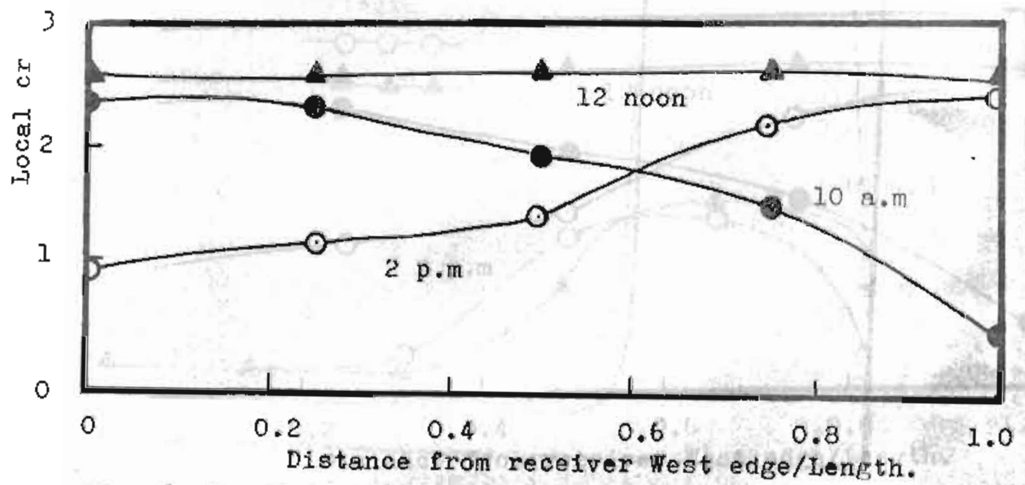

Fig. 6. Local concentration ratio in the receiver length direction at different hours.

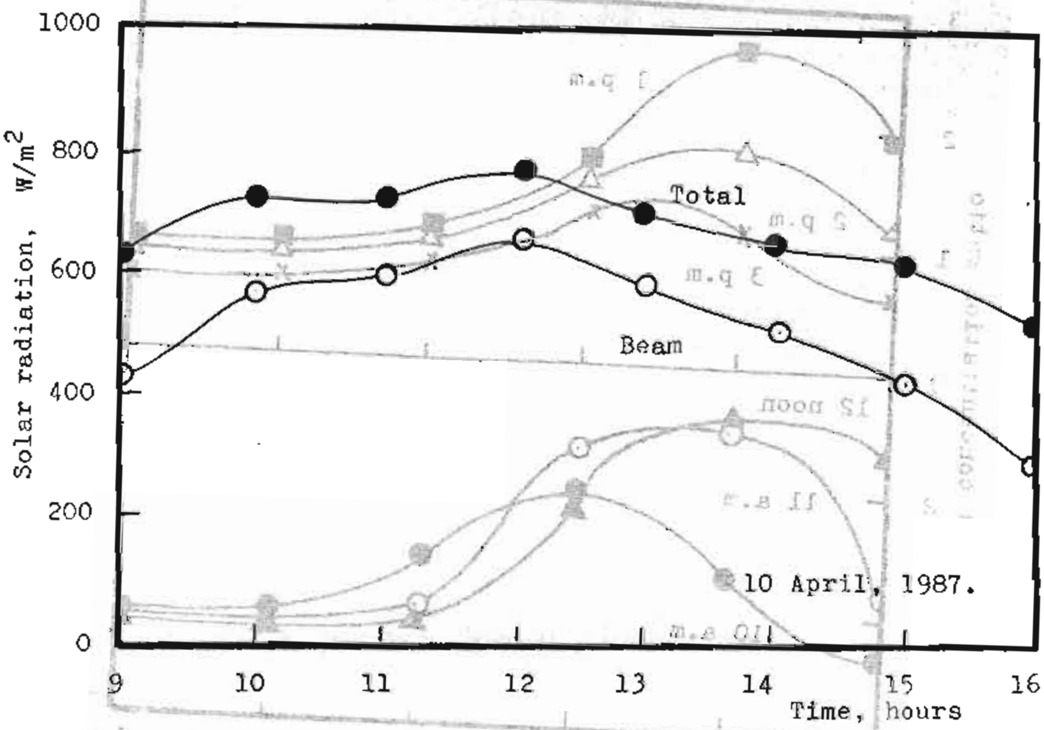

Fig. 7. Solar radiation data recoded on a horizontal plane.

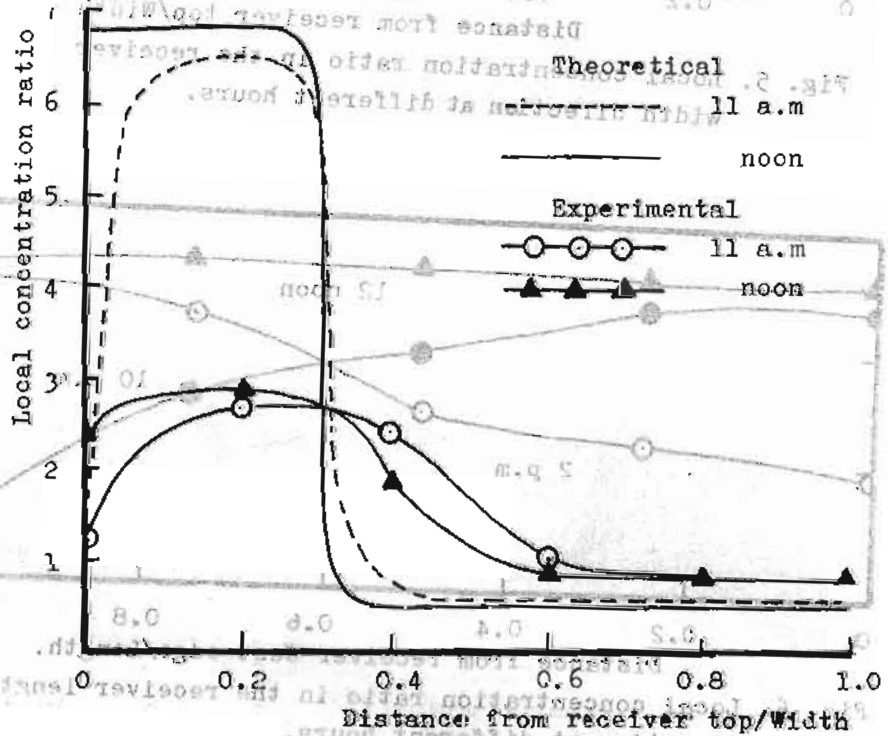

Fig. 8. Comparison between the measured and predicted local concentration ratios in the receiver width direction.

flux distribution over a flat receiver. By this design trend, the overheating problem can be avoided without any addetional costs. Results have shown that, experimental data on a prototype model are necessary to determine the receiver active area for thermal and photovoltaic applications. Also the concentrator length should be large enough, to minimize the optical end losses. The def ects of the reflecting surface and assembly errors, should be avoided in the construction of this concentrator type to inprove its optical caracterestics.

7. NOMENCLATURE

A	concentrator acceptance angle
B	distance from the receiver top point to the position of the re-reflected ray
CR	concentration ratio
CR_m	maximum concentration ratio
cr	local concentration ratio
E	angle between the reflected ray extension and the x-axis
F	flux distribution on the receiver surface
I_n	normal total radiation intensity
$I_{b,n}$	normal beam radiation intensity
I_d	horizontal diffuse radiation intensity
j	number of reflections
L_n	width of the segment number n
R	receiver width
R_o	uniformly illuminated portion on the receiver
r_n	distance between top point of segment number n and the receiver.
S_n	angle of the segment number n with respect to the x-axis
W	angle between the receiver and the y-axis
Z	zenith angle
Z_p	projection of zenith angle on the E-W vertical plane
α	an orintation mechanism
β	tilt angle of the receiver to the horizontal
γ	diffuse reflectance of the surrounding space
θ_n	incident angle of the reflected beam from segment number n on the recjver
δ	declination angle
ϕ	latitude of the place
σ	angle between the projection of the sun ray on the N-S vertical plane and its noon position
ρ	reflectivity of the mirror segment.

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