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OVERLAND FLOW CHARACTERISTICS USING KINEMATIC WAVE FORMULATION

خصائص السريان السطحى باستخدام الصياغه الحركيه للموجه

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خلاصه:
يهدف البحث الى دراسة خصائص السريان السطحى الذى ينتج عن أمطار منتظمة الكثافه على سطح مائل غير منفذ وذلك باستخدام الطريقه المميزه لحل المعا دلات الأحادية الأبعاد والتى تمثل السريان الغير مستقر الغير منتظم مع الأخذ فى الأعتبار تطبيق تقريب الموجه الكينماتيكيه على معادلة بقاء كمية الحركه. وقد تم دراسة تأثير متغيرات السريان السطحى على الهيدروجراف وأيضا على أقصى تصرف ناتج وزمن حدوث هذا التصرف وذلك فى حالة السريان المضطرب. وتشمل متغيرات السريان السطحى ميل وطول السطح وكذلك مقاومة السطح للسريان هذا بالأضافه الى الكثافه المنتظمه للأمطار. وتم استنباط هيدروجراف السريان السطحى باستخدام نظرية الموجه الكينماتيكيه مع مقارنته بهيدروجراف السريان السطحى من دراسه معمليه سابقه وكذلك بهيدروجراف السريان السطحى المستنتج من دراسه سابقه والتى أستخدمت فيها طريقة الفروق المحدوده لحل المعادله الديناميكيه الكامله لقانون بقاء كمية الحركه.

ABSTRACT
The characteristics of overland flow resulting from a uniform rainfall intensity on an impervious sloping plane are investigated. The characteristic solution of the one dimensional unsteady nonuniform flow equations with the kinematic wave approximation applied to the momentum equation is used. The influence of the different overland flow parameters on the synthesized hydrograph, the peak runoff discharge, and the time taken to reach equilibrium is studied for the case of turbulent flow. The overland flow parameters include the slope and the length of the plane, the surface resistance, and the uniform rainfall intensity. A dimensionless peak runoff hydrograph is determined and compared with both Izzard's experimental dimensionless hydrograph and the dimensionless hydrograph determined by the finite difference solution of the full dynamic form of the momentum equation.

INTRODUCTION

The hydrologic response of watersheds to rainfall has always been a subject of great importance in the sense that the surface runoff in the form of overland flow resulting from the rainfall excess finally flows into rivers or reservoirs. The unsteady nature of the overland flow over plane

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surfaces and which is described by the one-dimensional shallow water St. Venant equations have been numerically solved using numerous explicit and implicit finite difference schemes. The kinematic wave approximation of the shallow water equations with the method of characteristics as a solution technique has the advantage that it provides a direct analytical solution to predict the overland flow hydrograph. Two important features of the outflow hydrograph are of interest, first, the peak runoff and its time of occurrence, second, the effect of the relevant parameters on the peak runoff and its time of occurrence.

Muzik [8] experimentally investigated the application of the instantaneous unit hydrograph and the kinematic wave concepts to simulate the rainfall-runoff process. Morgali and Linsely [2] introduced an explicit finite difference scheme to synthesize the overland flow hydrograph. They used the continuity equation and the full dynamic form of the momentum equation to simulate overland flow. The influence of the overland flow parameters was investigated and an empirically fitted equation was developed to evaluate the time to reach equilibrium t_e as a function of the different overland flow parameters in the form:

$$t_e = 59.6\frac{L^{0.593}n^{0.605}}{i^{0.388}S_o^{0.38}} \quad \text{..........(1)}$$

in which L, n, i, and S_o are length of the overland flow plane, Manning coefficient of roughness, uniform rainfall intensity, and slope of the overland flow plane, respectively. Holden and Stephenson [12] used an improved four-point finite difference scheme to solve the kinematic equations for overland flow. Hjelmfelt [11] applied the kinematic wave approach to study the influence of time distribution of rainfall on peak discharge and time to reach equilibrium. Ragan and Duru [6] applied the method of characteristics to develop a nomograph based on the kinematic wave formulation to determine the time to equilibrium for overland flow.

In the present study, the influence of the different overland flow parameters on the rising runoff discharge hydrograph is investigated. The kinematic wave approximation is used to derive the basic differential equation representing overland flow and the method of characteristic is used as the solution procedure. A comparison is made between the predicted results for the peak runoff discharge and the time to reach equilibrium with the corresponding results obtained by Morgali [2] and Izzard's experimental data [1].

THEORETICAL CONSIDERATIONS

For the overland flow on the plane shown in figure 1, the one-dimensional flow equations for unsteady nonuniform free surface flow are given by the mass and momentum conservation [3] and [9] in the form:

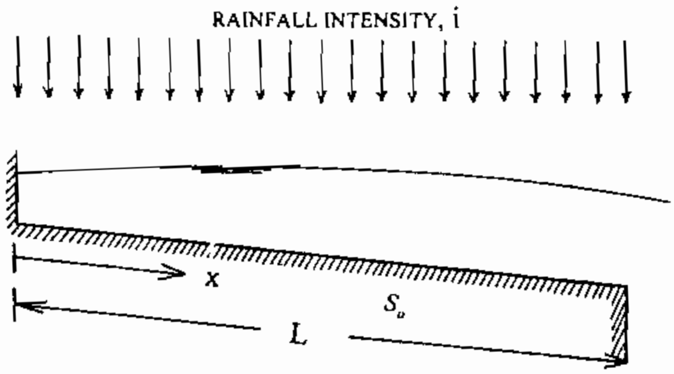

Figure 1. Definition sketch of the overland flow plane

$$\frac{\partial y}{\partial t}+\frac{\partial q}{\partial x}=i \qquad \text{.......(2)}$$

$$V\frac{\partial V}{\partial x}+\frac{\partial V}{\partial t}+g\frac{\partial y}{\partial x}=g(S_0-S_f) \qquad \text{.......(3)}$$

where t = time, x = distance along the overland flow plane, y = depth of flow, q = rate of flow per unit width, i = rainfall rate, V = average velocity, S_f = friction slope, S_0 = slope of the overland flow plane, and g = gravitational acceleration.

The kinematic wave approximation reduces the momentum equation to the form:

$$S_0 = S_f \qquad \text{.....(4)}$$

The friction slope S_f for unsteady nonuniform flow is assumed to be the same as that for uniform flow having the same depth and average velocity. Thus, for uniform turbulent flow, S_f can be expressed for wide channels from Manning equation as:

$$S_f = \frac{n^2 q^2}{2.208 y^{10/3}} \qquad \text{.....(5)}$$

From equation (5) , a depth discharge relationship can be written as:

$$q = \alpha y^m \qquad \text{.....(6)}$$

in which α and m are constants depending on the resistance equation used to express the friction slope term. For Manning's equation (5) α is $1.486 S^{0.5}/n$ and m is 5/3.

The dimensional kinematic wave formulation of overland flow comprise the continuity equation (2) and the momentum equation (6). Substitution of equation (6) into equation (2) yields:

$$\frac{\partial y}{\partial t} + \alpha m y^{m-1} \frac{\partial y}{\partial x} = i \qquad \text{.....(7)}$$

METHOD OF SOLUTION

The method of characteristics [3], [9], [11], and [12] is used to transform the partial differential equation (7) into an ordinary differential equation dy/dt = i and which is valid along the characteristic line defined by $dx/dt = \alpha m y^{m-1}$ in the (x-t) plane.

The rising hydrograph of the runoff discharge at the downstream boundary, x = L, can be expressed as:

$$q = \alpha (it)^m \qquad 0 \le t \le t_s \qquad \text{.....(8)}$$

where t_e, defined by the time to equilibrium, is the time at which the partial equilibrium hydrograph is reached and is given by:

$$t_e = \left(\frac{L i^{1-m}}{\alpha} \right)^{1/m} \quad \text{...(9)}$$

At that time, the runoff discharge reaches its peak value and a partial equilibrium (steady state condition) is reached for values of $t_e \le t \le T$ where T is the storm duration. Substitution of t in equation (8) equal to t_e from equation (9) provides the value of the peak runoff discharge.

ANALYSIS OF THE RESULTS AND COMPARISONS

The effect of the different overland flow parameters, namely, the length of the overland flow plane L, the slope of the overland flow plane S_o, the surface roughness n, and the rainfall intensity i, on the rising runoff hydrograph is shown in figure 2 a, b, c, and d. The effect of the surface roughness of the overland flow plane on the rising runoff hydrograph is shown in figure 2a for different values of Manning coefficient of roughness, n, varying from 0.02 to o.5. The other overland flow parameters have constant values of L = 72 ft., S_o = 0.04, and i = 3.66 in/hr. [2]. The figure shows that the roughness of the overland flow plane has no effect on the peak runoff but it only delays its time of occurrence as it increases.

The effect of the slope of the overland flow plane on the rising runoff hydrograph is shown in figure 2b. for different slopes varying from 0.01 to 0.06. The constant parameters are L =72 ft., n = 0.4, and i = 3.6 in/hr [2]. Similar to the roughness effect, the slope does not have any influence on the peak runoff but it only decreases its time of occurrence as it increases.

Figures 2e shows the effect of varying the rainfall intensity from 1.89 in/hr. to 4 in/hr. with the constant values for the other parameters as L = 72 ft., n = 0.4, and S_o = 0.04 [2]. The increase of the rainfall intensity increases the peak runoff and decreases the time to equilibrium.

The effect of the length of the overland flow plane on the unoff hydrograph is shown in figure 2d. Different lengths of 30 ft., 72 ft., 100 ft., 150 ft., and 200 ft. are used . The constant parameters are i = 3.6 in/hr., S_o = 0.04, and n = 0.4 [2]. The figure shows that the length variation has no effect on the hydrograph shape but rather increases both the peak runoff and the time to equilibrium as it increases.

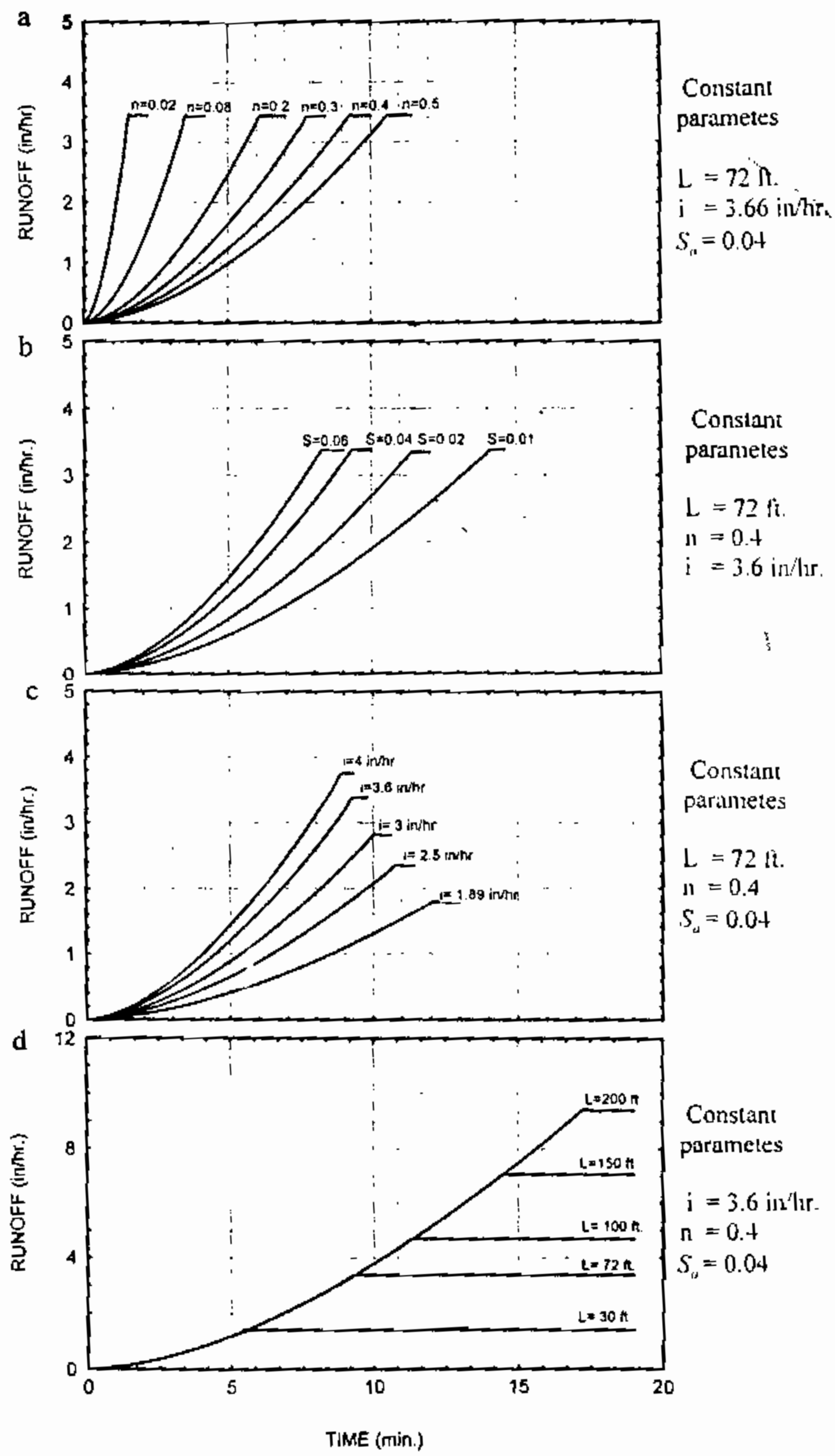

Figure 2. Effect of the different overland flow parameters on the rising runoff hydrograph.

Figure 3 illustrates the variation of the peak runoff with the different overland flow parameters. Both the kinematic wave approximation and the finite difference solution of the full dynamic equation [2] were used to compute the peak runoff and then both compared with the variation of each of the overland flow parameters.

Similar to figure 3, figure 4 is prepared to show the variation of the time to reach equilibrium with the different overland flow parameters. Also, both the kinematic wave approximation and the finite difference solution of the full dynamic equation [2] were used to compute the time to equilibrium using equation (1) and then both compared with variation of each of the overland flow parameters. Moreover, remarkable observations in figures 3 and 4 are evident, the kinematic wave approximation and the finite difference solution of the full dynamic equation show different effect of the surface roughness, the slope and length of the overland flow plane on the peak runoff dischrage while both show the same increasing effect of the rainfall intensity on the peak runoff discharge as the rainfall intensity increases. As far as the time of equilibrium is concerned, both the kinematic wave approximation and the finite difference solution of the full dynamic equation show the same effect of the different overland flow parameters on the peak runoff discharge.

As the time to reach equilibrium t_e represents the time at which the peak runoff discharge occurs and the flow conditions reaches its partial equilibrium state, i.e., the flow changes from unsteady nonuniform to steady uniform, it is then meaningful to study the effect of the kinematic wave approximation on the predicted values of the time to reach equilibrium against the respective values using the finite difference solution of the full dynamic equation. Such effect is shown in figure 5 which shows that the kinematic wave approximation resulted in

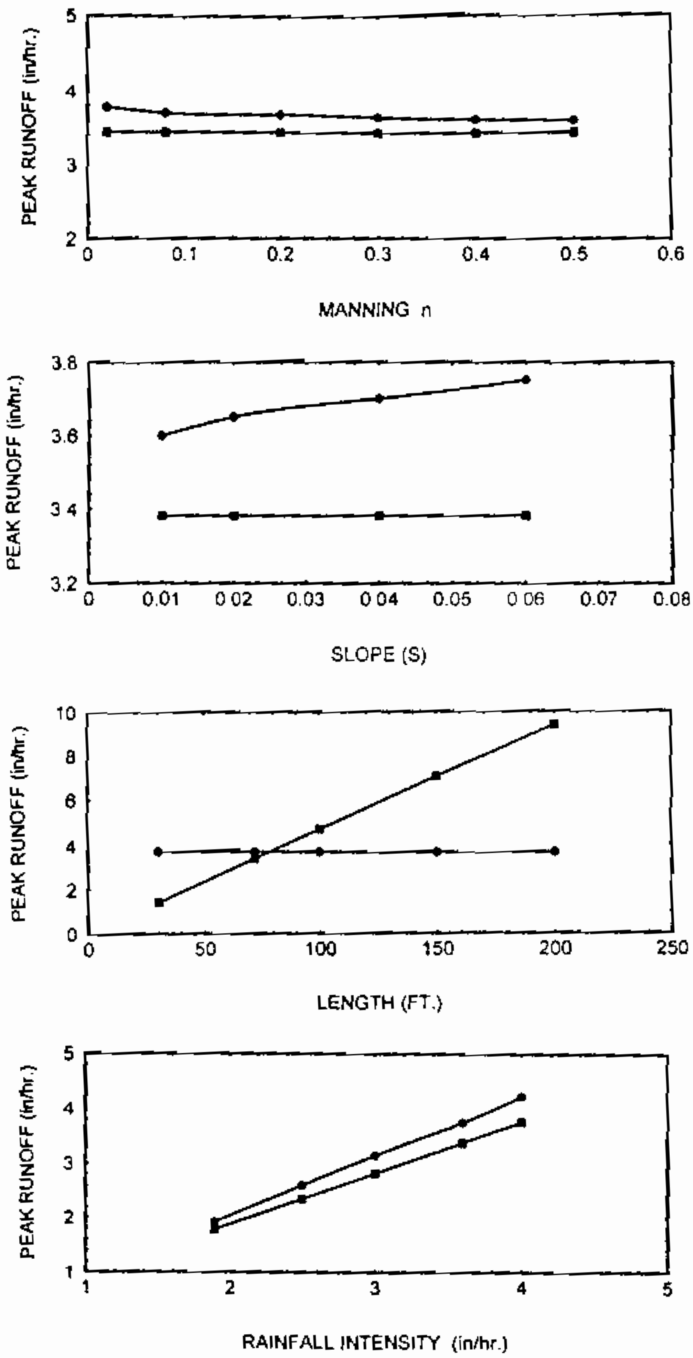

Figure 3. Effect of the kinematic wave approximation on the peak runoff for the different overland flow parameters.

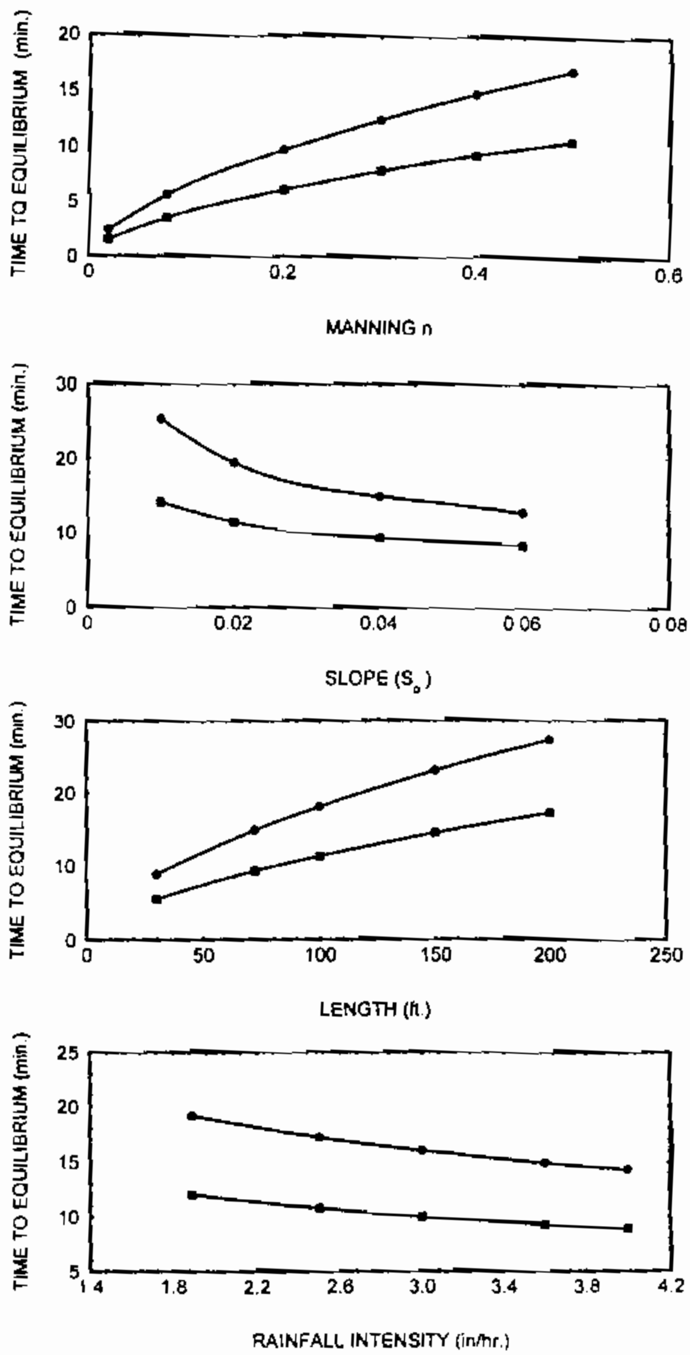

Figure 4. Effect of the kinematic wave approximation on the time to equilibrium for the different overland flow parameters.

a reduction in the estimated value of the time to equilibrium. Also, an increasing trend in the reduction is observed as the time to equilibrium increases.

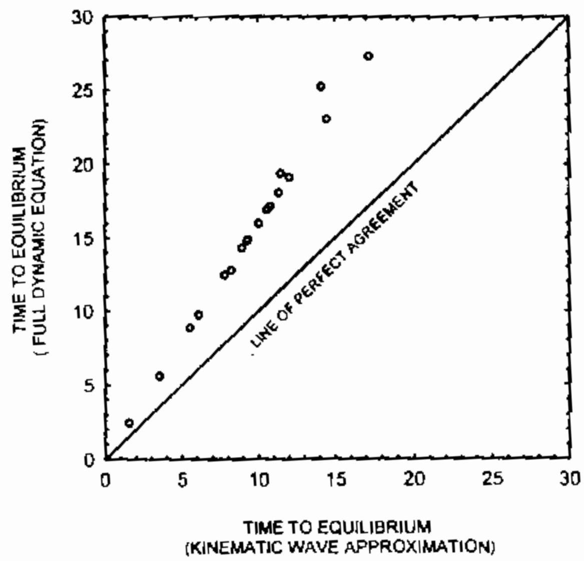

Figure 5: Effect of the kinematic wave approximation on the time to equilibrium.

The synthesized hydrograph can be represented in a dimensionless form by plotting (q/q_m) against (t/t_e) where q is the runoff in time t. Such plot is shown in figure 6. The dimensionless hydrograph based on the kinematic wave approximation is shown together with the dimensionless hydrographs based on both the finite difference solution of the full dynamic equation and the Izzard's experimental data [1]. The figure clearly illustrates the underestimation of the runoff based on the kinematic wave approximation comparable with the other dimensionless hydrographs.

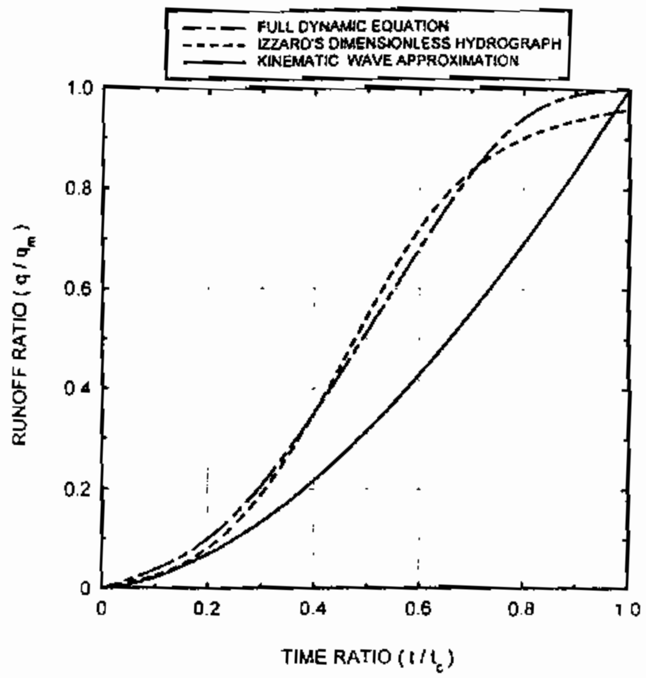

Figure 6: Dimensionless hydrographs (q/q_m) versus (t/t_c).

CONCLUSIONS

From the present study, the following conclusions are drawn:

1- The simulation of overland flow using the kinematic wave approach resulted in a reduction in the estimated values of the peak runoff discharge and the time to reach equilibrium.

2- The kinematic wave approach and the finite difference solution of the full dynamic equation show different effects of the surface roughness, slope and length of the overland flow plane on the peak runoff discharge.

3- The developed dimensionless hydrograph based on the kinematic wave approximation showed considerable deviation from both dimensionless hydrographs based on Izzard's experimental data and the finite difference solution of the full dynamic form of the momentum equation

4- Further verification to check the validity of the kinematic wave model to simulate overland flow is still needed by conducting a comprehensive comparisons with the numerous available explicit and implicit finite difference solutions of the full dynamic form of the momentum equation.

REFERENCES

1- Izzard, C. F., " Hydraulics of Runoff from Developed Surfaces," Proceedings, Vol. 26, Highway Research Board, National Research Councel, Washington D.C., 1946.

2- Morgali, J. R., and Linsely, R. K., " Computer Analysis of Overland Flow," Journal of the Hydraulics Division, ASCE, Vol. 91, No. HY3, May 1965.

3- Woolhiser, D.A., and Liggett, J.H.,"Unsteady One-dimensional Flow Over a Plane - The rising Hydrograph," Water Resources Research, Vol. 3, No. 3, 1967.

4- Henderson, F. M., " Flood Routing," Open Channel Flow, 3rd Ed., The Macmillan Co., New York, 1969.

5- Morgali, J. R.," Laminar and Turbulent Overland Flow Hydrographs, Journal of the Hydraulics Division, ASCE, Vol. 96, No. HY2, Feb. 1970.

6- Ragan, R. M., and Duru, J. O., " Kinematic Wave Nomograph for Times of Concentration," Journal of the Hydraulics Division, ASCE, Vol. 98, No. HY10, October 1972.

7- Shen, H. W. and Li, R. M., " Rainfall Effect on Sheet Flow Over Smooth Surface," Journal of the Hydraulics Division, ASCE, Vol. 99, No. HY5, 1973.

8- Muzik, I., " Laboratory Experiments with Surface Runoff," Journal of the Hydraulics Divisions, ASCE, Vol. 100, N0. HY4, April 1974.

9- Woolhiser,D. A., Simulation of Unsteady Overland flow, Chapter 12 in Unsteady Flow in Open Channels. Edited by K. Mahmood and V. Yevjevich, WRP, 1975.

10- Morris, E. M., and Woolhiser, D. A., " Unsteady One-dimensional Flow Over a Plane: Partial Equilibrium and Recession Hydrographs," Water Resources Researeh, Vol. 16, No. 2, April 1980.

11- Hjelmfelt, A., " Overland Flow from Time-Distributed Rainfall," Journal of the Hydraulics Division, ASCE, Vol. 107, No. HY2, brua February 1981.

12- Holden, A.P., and Stephenson, D., " Improved Four-Point Solution of the Kinematic Wave," Journal of Hydraulic Research, ASCE, Vol. 26, No. 4, 1988.

The following symbols are used in this paper:

g	= gravitational acceleration;
i	= rainfall intensity;
L	= length of the overland flow plane;
n	= Manning coefficient of roughness;
q	= runoff discharge per unit width;
q_m	= peak runoff discharge per unit width;
S	= slope of the overland flow plane;
S_f	= friction slope;
T	= storm duration;
t	= time;
t_e	= time to equilibrium;
V	= average velocity;
x	= distance along the overland flow plane;
y	= depth of flow; and
α and m	= constants.