INFILTRATION FROM BURIED PIPES IN UNSATURATED SOILS

التسرب من الأنابيب المطمورة فى التربة الغير مشبعة

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خلاصة : يتناول هذا البحث دراسة التسرب من الانابيب المطمورة على بعد معين من سطح التربة وتبعد مسافات متساوية عن بعضها البعض والتى تمثل احدى طرق الرى المعروفه .

معادلة التسرب غير الخطيه هى معادلة تفاضلية جزئية تصف التسرب فى كثير من المجالات العلمية كانتقال الحرارة والانتشار خلال الاوساط المسامية . ولما كانت هذه المعادلة لا يوجد لها حل تحليلى فقد استخدمت طريقة الاتجاه المتبادل الضمنية (ADI method) للحصول على وصف كمى وكيفى لحالة الرطوبة وموضع جبهة الابتلال فى الاتجاهين الافقى والرأسى عند أزمنة مختلفة .

وتتضمن الحل باستخدام طريقة الاتجاه المتبادل الضمنية تحويل المعادلة التفاضلية الجزئيه الى معادلة فروق وتحويل المعادلات الغير خطية الى معادلات خطية ثم حل المعادلات الخطية باستخدام طريقة الحذف لجاوس وقد أدمجت فى هذه المعادلات الشروط الابتدائيه والحدية لحالات التربة تحت الدراسة .

وتم اعداد وتنفيذ برنامج على الحاسب الآلى حيث امكن استنتاج المحتوى الرطوبى وموضع جبهة الابتلال فى منطقة التسرب فى الاتجاهين الرأسى والافقى عند ازمنة مختلفة .

ABSTRACT

The problem of the unsteady infiltration from buried pipes into unsaturated soils is solved numerically using the Alternating Direction Implicit (ADI) difference method. The water content distribution and the location of the wetting front can be predicted at any instant of time.

INTRODUCTION

Infiltration is an example of the general phenomenon of water movement in porous media. Unsteady infiltration from buried pipes is of practical importance in the fields of agriculture and engineering. In irrigation process, it is of great importance to determine the changes in water content and its distribution during the irrigation period. Furthermore, it is important from the practical point of view to know the position of the wetting front, since the excessive infiltration is not desirable when the vertical advance of the wetting front reaches the lower end of the root zone, which means a waste of water. Moreover, the development of the moisture profiles is essential to overcome drainage problems, and to control the moisture content in the soil.

The dynamics of the one-dimensional infiltration has been studied intensively [2],[3], but relatively less work has been devoted to the more complicated problems of two and three-dimensional systems. A general flow equation for moisture movement in unsaturated soil was developed by combining Darcy's law with the equation of continuity [2]. An iterative procedure using Boltzmann transformation was developed by Philip [3] to reduce the nonlinear diffusion equation obtained by [2] to a nonlinear ordinary differential equation. Philip [4] directed his study towards the two-dimensional problem of infiltration from a semicircular furrow and the three-dimensional problem of infiltration from a hemispherical cavity. The soil diffusivity was considered constant, thus the governing equations were changed into a linearized form. The problem of the two-dimensional transient transfer of water from rectangular unsaturated or partially unsaturated soil slabs was solved by [6] using the ADI difference method. Also the ADI method was adopted by [8] to solve the unsteady two-dimensional flow medium for unsaturated soils. The results of this study confirmed the validity of the ADI algorithm for solving the unsteady two-dimensional flow problems. Infiltration from a horizontal semi-cylindrical furrow into several soils and porous media, was investigated by Peck and Talsma [7]. They subjected their experimental findings to approximate solutions for commulative infiltration. The two-dimensional infiltration from a semicircular furrow into unsaturated soil was solved numerically by [9], who developed a numerical technique using the ADI method. The obtained numerical results were verified experimentally.

The problem of infiltration from buried pipes into unsaturated soils has been scarcely investigated. Therefore the present study is directed to this case and the ADI method is applied to investigate the following objectives:

i) finding the moisture content distribution in the soil,

ii) specifying the location of the wetting front as a function of time.

GOVERNING EQUATIONS

The continuity equation describing the moisture flow in unsaturated soils is given by

$$\partial\theta/\partial t = - \operatorname{div} V , \tag{1}$$

where θ is the volumetric moisture content, t is the time and V is the volumetric flux of water which is represented by its components:

$$V_x = -D(\theta)\, \partial\theta/\partial x ,$$

$$V_y = -D(\theta)\, \partial\theta/\partial y$$

and $$V_z = -D(\theta)\, \partial\theta/\partial z + K(\theta) , \tag{2}$$

where $D(\theta)$ is the diffusivity and $K(\theta)$ is the capillary conductivity of the soil.

Substituting equation (2) into (1), then

$$\partial\theta/\partial t = \partial/\partial x\ [\ D(\theta)\ \partial\theta/\partial x\ ] + \partial/\partial y\ [\ D(\theta)\ \partial\theta/\partial y\ ] + \partial/\partial z\ [\ D(\theta)\ \partial\theta/\partial z\ ] - \partial K(\theta)/\partial z. \qquad (3)$$

This equation is known as the flow equation.
For the two-dimentional infiltration we have

$$\partial\theta/\partial t = \partial/\partial x\ [\ D(\theta)\ \partial\theta/\partial x\ ] + \partial/\partial z\ [\ D(\theta)\ \partial\theta/\partial z\ ] - \partial K(\theta)/\partial z. \qquad (4)$$

MATHEMATICAL MODEL

A parallel set of equally spaced circular pipes is placed at an appropriate depth inside the soil, Figure (1). The pipes are oriented in the direction of the Y axis, which means that the flow is independent of the Y coordinate.

Due to symmetry, the soil medium can be subdivided into identical rectangular soil slabs. Each slab acts as an independent unit in the sense that there is no flow from one slab to another.

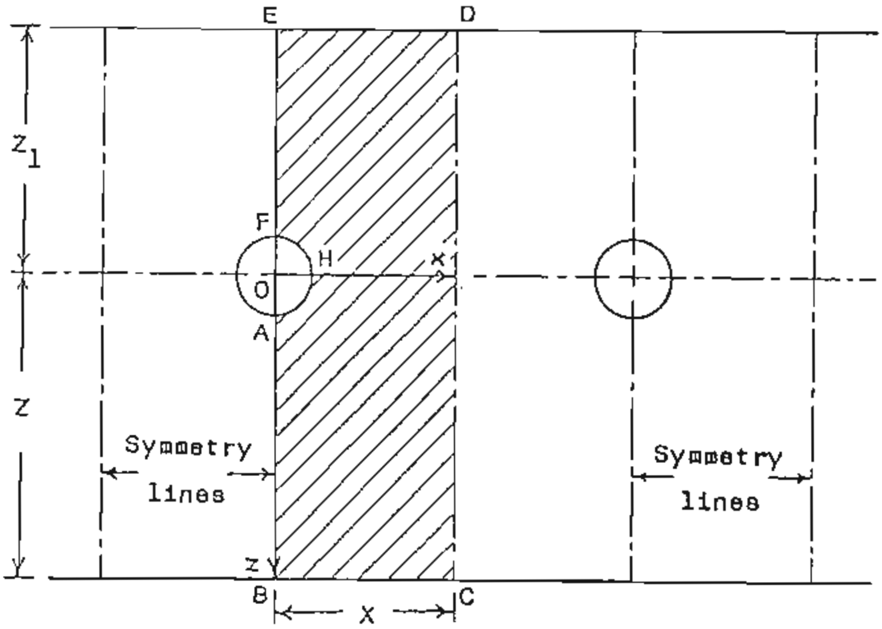

Fig.(1) Schematic diagram of a homogeneous soil irrigated by a set of circular buried pipes equally spaced at distances of 2X.

It is assumed that the initial moisture content in the soil is uniformly distributed and is sufficiently low. The infiltration through the slab is governed by equation (4) with the initial conditions:

$$\theta = \theta_1 \quad \text{for} \quad x^2 + z^2 > R^2 \quad \left| \begin{array}{c} 0 \le x \le X \\ -Z_1 \le z \le Z \end{array} \right| \quad , \; t = 0. \qquad (5)$$

Moreover, the boundary conditions are:

1- Along the semicircle AHF :

$$\theta = \theta_s \quad \text{for} \quad x^2 + z^2 = R^2 \quad , \quad t \ge 0 \; . \qquad (6)$$

2- Along the symmetry lines EB and DC :

$$\partial\theta/\partial x = 0 \quad \text{for} \quad \left| \begin{array}{ll} x = 0 & -Z_1 \le z \le -R \\ x = 0 & R \le z \le Z \\ x = X & -Z_1 \le z \le Z \end{array} \right| \; , \; t \ge 0 \; . \qquad (7)$$

3- Along the soil surface ED :

$$-D(\theta)\, \partial\theta/\partial z + K(\theta) = 0 \quad \text{for } z = -Z_1 \; , \; 0 \le x \le X \; , t \ge 0 \; . \qquad (8)$$

4- Along the line BC:

$$-D(\theta)\, \partial\theta/\partial z + K(\theta) = 0 \quad \text{for } z = Z \; , \; 0 \le x \le X \; , \; t \ge 0 \; . \qquad (9)$$

METHOD OF SOLUTION

The ADI technique approximates the partial differential equation (4) and the conditions (5) to (9) by means of difference equations. The obtained equations refer to a set of points of a rectangular grid with increments $\Delta x, \Delta z$ and $1/2\ \Delta t$ in the (x,z,t) space. The points are denoted by (j,k,n) where:

$$x = (j - 1)\,\Delta x \qquad , \qquad j = 0, 1, 2, \ldots\ldots, J \; ;$$

$$z = (k - K_1 - 1)\,\Delta z \qquad , \qquad k = 0, 1, 2, \ldots\ldots, K+K_1 \; ;$$

$$t = n\,\Delta t \qquad , \qquad n = 0, 1, 2, \ldots\ldots, N \; ;$$

knowing that $J = \text{entier}\,(X/\Delta x) + 2$, $K = \text{entier}\,(Z/\Delta z) + 2$

and $K_1 = \text{entier}\,(Z_1/\Delta z)$.

The values of the dependent variables at the grid point (j,k,n) in the flow medium is denoted by $\theta_{j,k}^{n}$. The domain boundaries $x = 0$, $x = X$, $z = -Z_1$ and $z = Z$ correspond to $j = 1$, $j = J-1$, $k = 1$ and $k = K+K_1-1$. Thus on all sides of the domain the grid is extended by one grid increment beyond the boundary of the domain. Zero and N values of time correspond to the beginning and the end of the infiltration period respectively.

In the ADI method each time step Δt is further subdivided into two different stages. During the first stage the solution is advanced from θ^n to an intermediate value $\theta^{n+\frac{1}{2}}$ by the following equation [1]:

$$\frac{\triangle\,(\theta^{n}_{j,k})}{1/2\ \triangle t} = \frac{[\delta(D(\theta)\ \delta\theta)]^{n+\frac{1}{2}}_{x}}{(\triangle x)^2} + \frac{[\delta(D(\theta)\ \delta\theta)]^{n}_{z}}{(\triangle z)^2} - \left[\frac{\partial k(\theta)}{\partial z}\right]^{n}. \quad (10)$$

The left-hand side of the partial differential equation (10) is evaluated by the forward difference operator $\triangle$ as

$$\triangle\,(\theta^{n}_{j,k}) = \theta^{n+\frac{1}{2}}_{j,k} - \theta^{n}_{j,k}\,, \quad (11)$$

while the first two terms of the right hand side of equation (10) are calculated by the central difference operator δ:

$$\frac{[\delta(D(\theta)\ \delta\theta)]^{n+\frac{1}{2}}_{x}}{(\triangle x)^2} + \frac{[\delta(D(\theta)\ \delta\theta]^{n}_{z}}{(\triangle z)^2} =$$

$$[1/(\triangle x)^2][D(\theta^{n+\frac{1}{2}}_{j+\frac{1}{2},k})(\theta^{n+\frac{1}{2}}_{j+1,k}-\theta^{n+\frac{1}{2}}_{j,k}) - D(\theta^{n+\frac{1}{2}}_{j-\frac{1}{2},k})(\theta^{n+\frac{1}{2}}_{j,k}-\theta^{n+\frac{1}{2}}_{j-1,k})]$$

$$+[1/(\triangle z)^2][D(\theta^{n}_{j,k+\frac{1}{2}})(\theta^{n}_{j,k+1}-\theta^{n}_{j,k}) - D(\theta^{n}_{j,k-\frac{1}{2}})(\theta^{n}_{j,k} - \theta^{n}_{j,k-1})]. \quad (12)$$

The last term on the right-hand side of equation (10) is approximated using Taylor series expansion. Neglecting all terms of the second and higher powers of $\triangle z$, we get

$$\left|\frac{\partial(k(\theta)}{\partial z}\right|^{n}_{j,k} = [1/(2\triangle z)][K(\theta^{n}_{j,k+1} - K(\theta^{n}_{j,k-1})]. \quad (13)$$

Substitution from (11), (12) and (13) into (10) gives

$$\frac{\theta^{n+\frac{1}{2}}_{j,k} - \theta^{n}_{j,k}}{(1/2)\triangle t} = \frac{1}{(\triangle x)^2}\,[D(\theta^{n+\frac{1}{2}}_{j+\frac{1}{2},k})(\theta^{n+\frac{1}{2}}_{j+1,k} - \theta^{n+\frac{1}{2}}_{j,k})$$

$$-D(\theta^{n+\frac{1}{2}}_{j-\frac{1}{2},k})(\theta^{n+\frac{1}{2}}_{j,k} - \theta^{n+\frac{1}{2}}_{j-1,k})]$$

$$+ \frac{1}{(\triangle z)^2}\,[D(\theta^{n}_{j,k+\frac{1}{2}})(\theta^{n}_{j,k+1} - \theta^{n}_{j,k})$$

$$-D(\theta^{n}_{j,k-\frac{1}{2}})(\theta^{n}_{j,k} - \theta^{n}_{j,k-1})]$$

$$-[1/(2\triangle z)][K(\theta^{n}_{j,k+1}) - K(\theta^{n}_{j,k-1})]. \quad (14)$$

This equation is a finite difference approximation of the differential equation (4). The boundary conditions to be satisfied by $(\theta)^{n+½}$ in equations (6) and (7) can be written in the finite difference form. They are respectively:

$$\theta_{j,k}^{n+½} = \theta_s \qquad \text{for } j = \text{entier } (x / \triangle x) + 1 ,$$

$$x = [\, R^2 - ((k - 1)\triangle z)^2 \,]^{½}$$

$$\text{and } k = K_1-k_s+1, K_1-k_s+2, .., K_1, .., K_1+k_s, K_1+k_s+1;$$

$$k_s = \text{entier } (z/\triangle z) + 1, \qquad (15)$$

$$\theta_{0,k}^{n+½} = \theta_{2,k}^{n+½} \qquad \text{for } k = 1, 2, 3, ...K_1 - k_s$$

$$\text{and } k = K_1+k_s+2,\ K_1+k_s+3, ...,\ K+K_1-2,\ K+K_1-1$$

$$\text{and } \theta_{J,k}^{n+½} = \theta_{J-2,k}^{n+½} \qquad \text{for } k = 1, 2, 3, ..., K+K_1-2, K+K_1-1. \qquad (16)$$

The finite difference equation (14) is implicit only in the x direction in which $\theta^{n+½}$ is unknown; it is also explicit in the z direction in which θ^n is known. It is clear that equation (14) is non-linear since the value of $D(\theta_{j\pm½,k}^{n+½})$ depend on $\theta_{j\pm½,k}^{n+½}$ for which solutions are being sought. The arguments $\theta_{j\pm½,k}^{n+½}$ of the funtion $D(\theta_{j\pm½,k}^{n+½})$ in equation (14) are approximated by means of the following explicit difference equation based on equation (4):

$$\frac{\theta_{j,k}^{n+½} - \theta_{j,k}^{n}}{(1/2)\triangle t} = \frac{1}{(\triangle x)^2} \{D(\theta_{j+½,k}^{n})(\theta_{j+1,k}^{n} - \theta_{j,k}^{n})$$

$$- D(\theta_{j-½,k}^{n})(\theta_{j,k}^{n} - \theta_{j-1,k}^{n})\}$$

$$+ \frac{1}{(\triangle z)^2} \{D(\theta_{j,k+½}^{n})(\theta_{j,k+1}^{n} - \theta_{j,k}^{n})$$

$$- D(\theta_{j,k-½}^{n})(\theta_{j,k}^{n} - \theta_{j,k-1}^{n})\}$$

$$- \frac{1}{2\,\triangle z} \{K(\theta_{j,k+1}^{n}) - K(\theta_{j,k-1}^{n})\}. \qquad (17)$$

Assuming a square grid, i.e. $\triangle x = \triangle z$ and putting

$$\alpha = \frac{\Delta t}{2(\Delta x)^2} = \frac{\Delta t}{2(\Delta z)^2} \quad , \quad \beta = \frac{\Delta t}{4(\Delta z)}$$

and rearranging equation (17), the following equation is obtained:

$$\theta_{j,k}^{n+½} = \theta_{j,k}^{n} + \alpha \{D(\theta_{j+½,k}^{n})(\theta_{j+1,k}^{n} - \theta_{j,k}^{n}) - D(\theta_{j-½,k}^{n})(\theta_{j,k}^{n} - \theta_{j-1,k}^{n})$$
$$+ D(\theta_{j,k+½}^{n})(\theta_{j,k+1}^{n} - \theta_{j,k}^{n}) - D(\theta_{j,k-½}^{n})(\theta_{j,k}^{n} - \theta_{j,k-1}^{n})\}$$
$$- \beta \, [K(\theta_{j,k+1}^{n}) - K(\theta_{j,k-1}^{n})]. \tag{18}$$

Equation (18) enables the calculation of the predictor $\theta_{j,k}^{n+½}$ directly directly from the previous distribution $\theta_{j,k}^{n}$. Equation (14) can be transformed into a more convenient form using elementary algebraic manipulation which gives the following equation:

$$A_{j,k}^{n} \, \theta_{j-1,k}^{n+½} + B_{j,k}^{n} \, \theta_{j,k}^{n+½} + C_{j,k}^{n} \, \theta_{j+1,k}^{n+½} = E_{j,k}^{n} \, , \tag{19}$$

where $A_{j,k}^{n} = -\alpha \; D(\theta_{j-½,k}^{n+½})$,

$B_{j,k}^{n} = 1 + \alpha \; D(\theta_{j-½,k}^{n+½}) + \alpha D(\theta_{j+½,k}^{n+½})$,

$C_{j,k}^{n} = -\alpha \; D(\theta_{j+½,k}^{n+½})$

and

$$E_{j,k}^{n} = \alpha \; D(\theta_{j,k-½}^{n}) \; \theta_{j,k-1}^{n} + \{1 - \alpha \; D(\theta_{j,k-½}^{n})$$
$$- \alpha \; D(\theta_{j,k+½}^{n})\} \; \theta_{j,k}^{n} + \alpha \; D(\theta_{j,k+½}^{n}) \; \theta_{j,k+1}^{n}$$
$$- \beta \; [K(\theta_{j,k+1}^{n}) - K(\theta_{j,k-1}^{n})] \; .$$

For fixed k the system of linear equations (19) and (16) with (15) forms a closed system of equations with equal number of unknowns $\theta_{j,k}^{n+½}$. This system is independent of the other systems corresponding to different values of k. The solution of the system k represents the distribution of the moisture content along the horizontal line k of the grid. Varying k from 1 to $(K+K_1-1)$, the values of $\theta_{j,k}^{n+½}$ can be obtained at all nodes of the grid at the end of the first stage corresponding to $t = (n+½)\Delta t$.

During the second stage of the ADI time step, the solution is advanced from $\theta^{n+½}$ to θ^{n+1} by the following finite difference approximation:

$$\frac{\theta_{j,k}^{n+1} - \theta_{j,k}^{n+½}}{(1/2)\Delta t} = [1/(\Delta x)^2] \{ D(\theta_{j+½,k}^{n+½})(\theta_{j+1,k}^{n+½} - \theta_{j,k}^{n+½})$$

$$- D(\theta_{j-½,k}^{n+½})(\theta_{j,k}^{n+½} - \theta_{j-1,k}^{n+½})\}$$

$$+ [1/(\Delta z)^2][D(\theta_{j,k+½}^{n+1})(\theta_{j,k+1}^{n+1} - \theta_{j,k}^{n+1})$$

$$- D(\theta_{j,k-½}^{n+1})(\theta_{j,k}^{n+1} - \theta_{j,k-1}^{n+1})]$$

$$-[1/2(\Delta z)][K(\theta_{j,k+1}^{n+1}) - K(\theta_{j,k-1}^{n+1})] \quad (21)$$

The boundary conditions to be satisfied by θ^{n+1} are (6), (8) and (9). In the finite difference form these become respectively:

$$\theta_{j,k}^{n+1} = \theta_s \quad \text{for } k = K1 \pm \text{entier } (z/\Delta z) + 1,$$

$$z = [R^2 - ((j-1)\Delta x)^2]^{½}$$

$$\text{and } j = 1,2,3,\ldots,j_s\ ;$$

$$j_s = \text{entier } (R/\Delta x) + 1, \quad (22)$$

$$\theta_{j,0}^{n+1} = \theta_{j,2}^{n+1} \quad \text{for } j = 1, 2, \ldots, J-1 \quad (23)$$

$$\text{and } \theta_{j,K+K1}^{n+1} = \theta_{j,K+K1-2}^{n+1} \quad \text{for } j = 1, 2, \ldots, J-1. \quad (24)$$

The finite difference approximation (21) is implicit in the z direction in which θ^{n+1} is unknown, it is also explicit in the x direction in which $\theta^{n+½}$ is known. The argument $\theta_{j,k\pm½}^{n+1}$ of the functions $D(\theta_{j,k\pm½}^{n+1})$ and $K(\theta_{j,k\pm½}^{n+1})$ in (21) are predicted by an explicit difference equation similar to (17):

$$\theta_{j,k}^{n+1} = \theta_{j,k}^{n+½} + \alpha\ [D(\theta_{j+½,k}^{n+½})(\theta_{j+1,k}^{n+½} - \theta_{j,k}^{n+½})$$

$$- D(\theta_{j-½,k}^{n+½})(\theta_{j,k}^{n+½} - \theta_{j-1,k}^{n+½}) + D(\theta_{j,k+½}^{n+½})(\theta_{j,k+1}^{n+½} - \theta_{j,k}^{n+½})$$

$$- D(\theta_{j,k-½}^{n+½})(\theta_{j,k}^{n+½} - \theta_{j,k-1}^{n+½})] - \beta[K(\theta_{j,k+1}^{n+½}) - K(\theta_{j,k-1}^{n+½})] \quad (25)$$

where α and β have the same meaning as defined above. Equation (25) can be solved for $\theta_{j,k}^{n+1}$ directly from the computed values of $\theta^{n+½}$.

Again equation (21) can be transformed into a more compact form by elementary algebraic transformation which gives the following equation:

$$A_{j,k}^{n+½}\ \theta_{j,k-1}^{n+1} + B_{j,k}^{n+½}\ \theta_{j,k}^{n+1} + C_{j,k}^{n+½}\ \theta_{j,k+1}^{n+1} = E_{j,k}^{n+½}, \qquad (26)$$

where

$$A_{j,k}^{n+½} = -\ \alpha\ D(\theta_{j,k-½}^{n+1}), \qquad (27)$$

$$B_{j,k}^{n+½} = 1 + \alpha\ D(\theta_{j,k-½}^{n+1}) + \alpha\ D(\theta_{j,k+½}^{n+1}), \qquad (28)$$

$$C_{j,k}^{n+½} = -\ \alpha\ D(\theta_{j,k+½}^{n+1}) \qquad (29)$$

and

$$E_{j,k}^{n+½} = \alpha\ D(\theta_{j-½,k}^{n+½})\ \theta_{j-1,k}^{n+½} + [1 - \alpha\ D(\theta_{j-½,k}^{n+½}) - \alpha\ D(\theta_{j+½,k}^{n+½})]\ \theta_{j,k}^{n+½} + \alpha\ D\ (\theta_{j+½,k}^{n+½})\ \theta_{j+1,k}^{n+½} - \beta\ [K(\theta_{j,k+1}^{n+1}) - K(\theta_{j,k-1}^{n+1})]. \qquad (30)$$

For each fixed value of j the system of linear equations (26), (22), (23) and (24) constitutes a closed system of linear algebraic equations of equal number of unknowns $\theta_{j,k}^{n+1}$. Each system is independent of the other systems corresponding to different values of j. The solution of the system j gives the values $\theta_{j,k}^{n+1}$ along the vertical line of the grid. Thus changing j from 1 to (J-1) yields the distribution at all nodes of the grid.

Thus the full cycle of the ADI method is completed. The whole process is iteratively repeated for θ^{n+2}, θ^{n+3}, ...and so on.

The systems of equations (19) and (26) can be written in matrix form as:

$$A\ \theta = E \qquad (31)$$

where A is a tridiagonal real matrix and θ E denote the associated real column vectors.

Equation (31) can be written in an explicit form, which can be easily solved by an adaptation of the Gaussian algorithm.

RESULTS AND DISCUSSION

This method was applied to a sandy clay loam soil whose diffusivity and capillary conductivity are determined experimentally and fitted by the following expressions:

$$D(\theta) = 64.865 \times 10^{-6} \exp(23.3858\ \theta) \quad \text{for} \quad \theta_i \le \theta \le \theta_s$$

and

$$K(\theta) = 64.5490 \times 10^{-13} \exp(64.8697\ \theta - 43.7416\ \theta^2) \quad \text{for } \theta_i \le \theta \le \theta_s$$

where θ_i is the initial water content
and θ_s is the saturation water content.

In the step-wise numerical solution, one encounters the problems of construction of finite difference systems, their method of solution, their stability, their covergence and their accuracy.

It was found that the implicit alternating direction equations arising from equation (4) are unconditionally stable for all values of ½ Δt, Δx and Δz [8]. Constant time steps Δt and space steps Δx and Δz are involved in the stability (convergence) criteria. For sandy clay loam soil several step sizes of ½ Δt =0.25, 0.5 and 1.0 min. and $\Delta x = \Delta z$ = 1.0, 2.0, and 4.0 cm. were tested for convergence. It was found that for ½ Δt = 0.5 min. and $\Delta x = \Delta z$ = 2.0 cm. convergence occurred and the calculated water content distribution at each node for the (n+1) time steps was similar to that calculated for ½ Δt = 0.25 min. and $\Delta x = \Delta z$ = 2.0 cm.(within a variation of 0.04% in water content).

The geometrical configuration of the flow medium is shown in Figure (1). The dimensions are chosen as: Z_1 = 40 cm., X = 45 cm. and Z = 50 cm. The origin (0,0) of the coordinates was placed at the centre of the pipe; a pipe of radius 5 cm. was considered.

Figure (2) shows the water content distribution during the two-dimensional infiltration from the buried pipe into the sandy clay loam soil. The water content distribution is given for a series of time at 119, 303 and 567 minutes. The wetting front (lines of θ taken arbitrarily as θ = 0.20 cm^3/cm^3) which separates the initial water content from higher water contents in the flow medium progresses in different directions with time at decreasing rate.

CONCLUSION

A computer program was developed to solve the two-dimensional flow equation of water infiltration from buried pipes into unsaturated soils by the modified ADI method, where the variable domain of solution is enlarged according to the propagation of the wetting front. The water content distribution was obtained at different series of time.

The influence of the gravitational term on the water content distribution in the flow medium was detected. The vertical advance of the lines of iso-water content in the positive z direction is greater than he horizontal advance in the x direction which exceeds the vertical advance in the negative z direction. This is due to the gravitational effect, as the flow equation (4) contains the term $\partial k/\partial z$. The horizontal flow is considered as a simple diffusion, while the vertical one is a superposition of two components, one of which is the gravitational flow due to the vertical gravitational potential gradient and the other is the diffusion downwards due to the gradient of the water content, [2] and [6].

The results show also that the lines of iso-water content close to the water source are wide apart compared with those close to the wetting front. Thus the moisture gradients are steeper close to the wetting front than those near the water source. The steep water content gradient at the wetting front is caused by the strong dependence of the capillary conductivity (and hence diffusivity) on water content.

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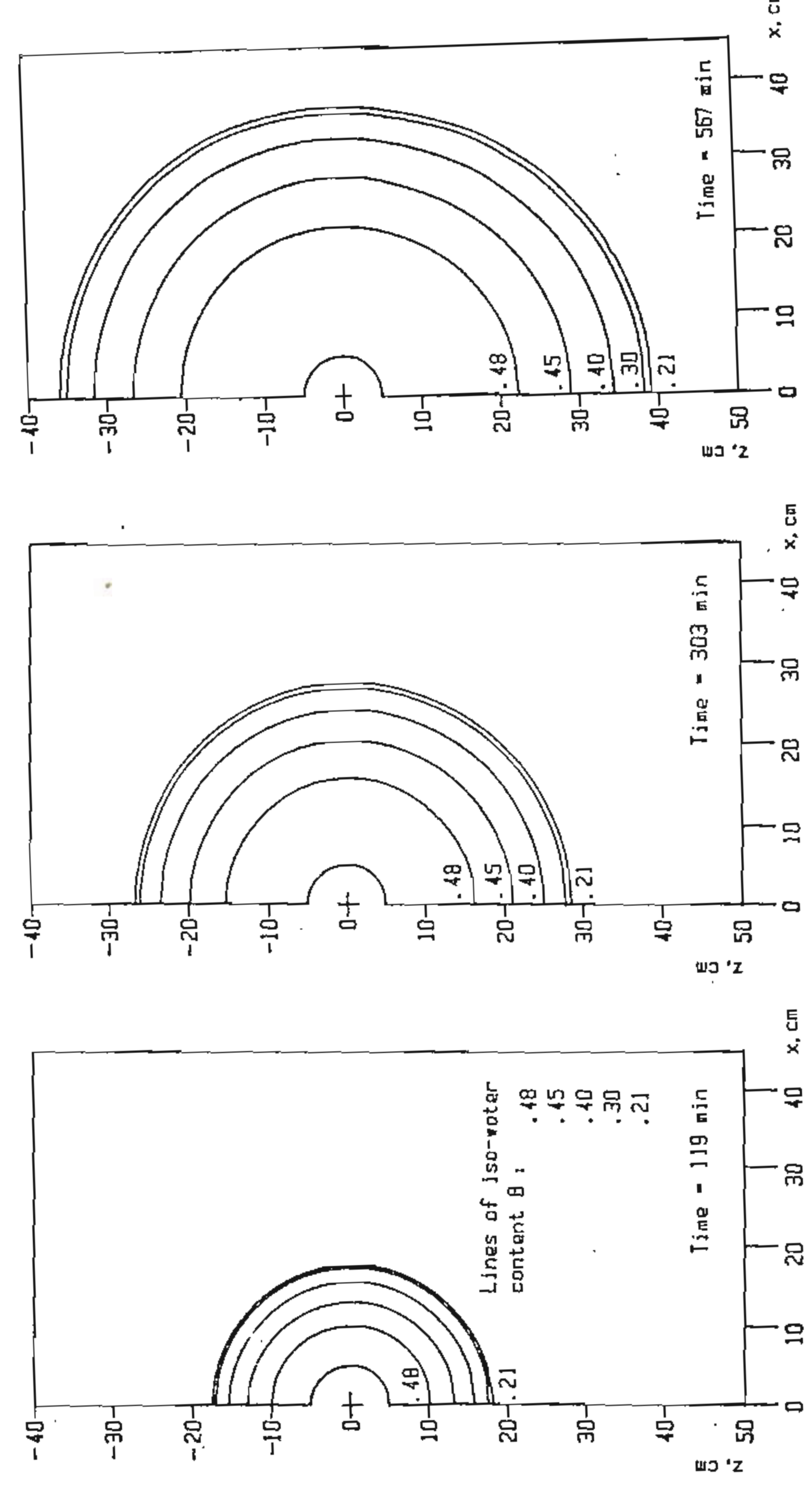

Fig.(2) Moisture content θ at different times.