

Multi-Objective Management of Heterogeneous Coastal Aquifers

إدارة أهداف مختلفة لخزانات جوفية ساحلية غير متجانسة

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خلاصة

يهدف هذا البحث بالحصول على افضل إدارة للخزانات الجوفية وذلك مع تعظيم الاستفادة المرجوة من هدف محدد أو مجموعة من الأهداف المتضاربة. تلك المشكلة معقدة وغير خطية ومقيدة. ولذلك تم استخدام الطريقة الحسابية الجينية للوصول لأحسن حل. وقد استخدمت طريقة العناصر المحددة لتمثيل سريان المياه الجوفية وذلك تحت ظروف سريان مستقر داخل خزان جوفي غير متجانس. واستغلت معادلة ستراك لتحويل المعادلة غير الخطية المتكيفة في السريان إلى معادلة خطية. كما استخدمت طريقة LU التفكيكية وذلك لقلب المصفوفة الرئيسية بالنموذج بغرض الحد من الزمن المطلوب للحسابات. واستخدمت طريقة القيود لتحويل الأهداف المختلفة لهدف واحدة. وقد تم كتابة برنامج كمبيوتر لتطبيق النموذج السابق. وحقق البرنامج المكتوب من خلال مشكلة لها حل تحليلي تم استخدام بعد ذلك لحل أمثلة افتراضية.

Abstract

A nonlinear optimization problem based on both single and multiple objective management functions for sustainable utilization of heterogeneous coastal aquifers was formulated and solved. The sharp interface approach was assumed within this work. Finite Element Method (FEM) was applied on Strack's formulation to simulate the hydraulic response of the steady-state heterogeneous aquifer under different objectives. The LU-decomposition method was exploited to inverse the conductance matrix, that enhanced the computation time. The constraint technique was used to simplify, without any approximation, the multi objective management problem to single objective management problem. Genetic Algorithm (GA) was used to solve the nonlinear optimization problem. A Fortran program was written and verified against a problem that has an analytical solution. A hypothetical example was solved for one and three objective functions. These functions are maximum magnitude of extracted freshwater, minimum drop in water table level, and minimum increment in volume of intruded saltwater.

Key Words: *Multiple Objectives- Coastal Heterogeneous Aquifer- Genetic Algorithm- Sharp Interface- Steady Flow- Finite Element Method- Strack's Formulation.*

1-Introduction

Due to the increased extracting of freshwater from coastal aquifers, saltwater intrusion may causes serious impacts in terms of environmental and economic impacts. To extract maximum amount of freshwater without salinization, a powerful optimization technique must be applied to catch best strategy. In this research Genetic Algorithm method (GA) was used. GA can handle easily a nonlinear constrained objective function without

need to its derivatives. Single and multiple objective functions were applied here, that represent optimal extraction of freshwater, minimum increment of intruded sea water volume and, minimum drop in water table level. The control variables were the pumping rates from the wells and the constraints were upper and lower pumping limits of each well. To evaluate the proposed objective functions, a simulation model is

needed. Several methods have been developed to analyze, predict, or control the seawater intrusion and to manage coastal aquifers, based on two approaches to simulate the saltwater-freshwater interface. In first approach that named by the sharp interface approach, both fresh and salt water zones are separated by a distinct sharp interface [1, 2]. Consequently there is no need to deal with the spatial distribution of concentration. Two flow equations must be solved simultaneously, one for freshwater and the other for seawater. The second approach is to consider the transition zone between freshwater and seawater using a density-dependent model of groundwater flow and salt transport [2, 3, 4]. In this case the concentration varies continuously over space.

In 1976, Strack presented a linear formulation. He used the Dupuit's Forchheimer assumption for dynamics of freshwater flow under steadiness condition in combination with static (stagnant) seawater [5, 6]. Ghyben-Herzberg approximation was used with Strack's formulation to specify the locus of the sharp interface, between the freshwater and the seawater zones. That formulation was used in this work instead of the nonlinear flow equations, that needs a special algorithm for tracking the sharp interface at the impervious bed [2].

To handle any practical aquifer, different hydraulic conductivities representing different zones of that aquifer must be measured. Generally most of the aquifers are heterogeneous. The goal of the present paper is to find optimal solution of different objective functions for a heterogeneous coastal aquifer using the Finite Element method (FEM) for simulating the

aquifer in combination with GA for the optimization procedure.

In spite of the great amount of researches concerned with multiple management and remediation of ground water, only few papers designated to coastal aquifers. Das [7] used the embedding technique through a gradient optimization method with the density dependent approach to study various conflicting objective functions under transient condition. Hallaji [8], used seven groundwater management functions to determine optimal planning of coastal aquifer in southern Turkey. He adopted the sharp interface approach to simulate the aquifer.

2- Problem Formulation

Strack derived a single potential theory such that a single linear equation could be applied to both of the saltwater and the freshwater zones under steady condition, see [5, 6], as:

$$\frac{\partial}{\partial x} \left(K \frac{\partial \phi}{\partial x} \right) + \frac{\partial}{\partial y} \left(K \frac{\partial \phi}{\partial y} \right) + R + \sum_{i=1}^{nw} \delta(x - x_i) Q_i \delta(y - y_i) = 0.0 \quad (1)$$

where, ϕ is the potential function (L^2), K is the hydraulic conductivity (L/T), x and y are rectangular coordinates (L), Q_i is pumping or recharging rate at well i (L^3/T), $\delta(z)$ is Dirac delta equal to 1 if z is zero otherwise equal to 0.0, R is rate of water infiltrating from rain fall (L/T) (for unconfined aquifers only), and nw is number of wells through the studied domain. It is of interest to remark that once the well is pumping saltwater, the static saltwater assumption used to calculate interface location is violated.

While Strack assumes a constant hydraulic conductivity in his

calculations, his formulation can handle aquifers that have spatially variable and anisotropic hydraulic conductivities.

Figures 1a and 1b give a definition sketch in the vertical cross-section of confined and unconfined aquifers, respectively. Distinction has been made between two zones, a freshwater zone (zone I) and a freshwater-saltwater zone (zone II). Strack demonstrated that for a homogenous, isotropic aquifer with horizontal impervious bed, a potential ϕ , which is continuous across the two zones, can be defined as, [5]:

$$\phi = Bh + \frac{(\rho_s - \rho_f)B^2}{2\rho_f} - (\rho_s / \rho_f)Bd \quad \text{for zone I} \quad (2)$$

$$\phi = \frac{\rho_f}{2(\rho_s - \rho_f)} \left[h + \frac{(\rho_s - \rho_f)}{\rho_f} (B-d) + d \right]^2 \quad \text{for zone II} \quad (3)$$

For unconfined aquifers:

$$\phi = \frac{1}{2} [h^2 - (\rho_s / \rho_f)d^2] \quad \text{for zone I} \quad (4)$$

$$\phi = \frac{\rho_f}{2(\rho_s - \rho_f)} [h-d]^2 \quad \text{for zone II} \quad (5)$$

For confined aquifers:

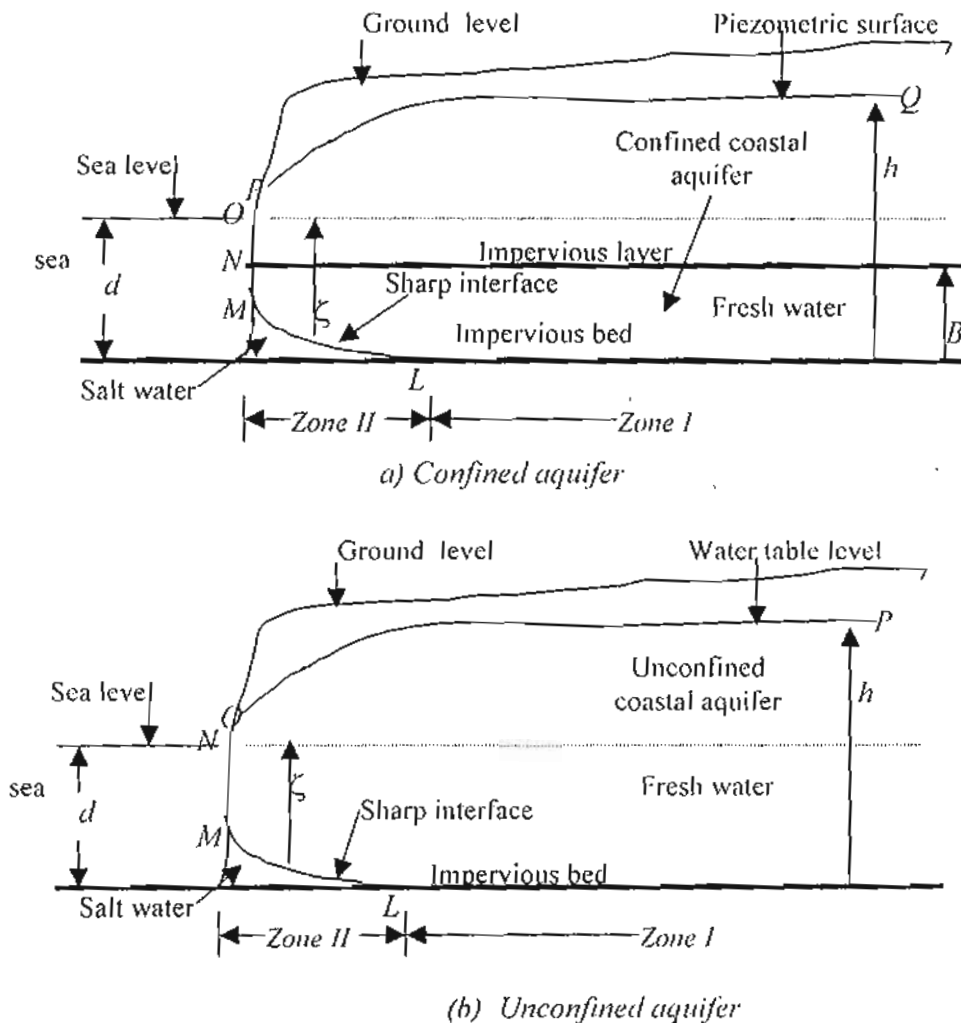


Figure 1. Definition sketch of saltwater intrusion in a) confined aquifer, and b) unconfined aquifer.

calculated from the numerical model reaches to 99.24% of optimal pumping achieved by the analytical solution, that can be considered sufficiently satisfactory. From the present results it can be noticed that GA can reach very close to the global minimum. To assure catching the optimal solution another gradient optimization method must be applied on final fittest individuals [18], or by repeating the solution process with different seed numbers.

7- Case Study I: (Single Objective Function)

In this example a heterogeneous unconfined aquifer was studied to find optimal pumping rate from a preexisting well system. The following parameters were used: $d=15.0m$, $n_w = \text{number of wells} = 15$, $YD=5500m$, $q_u = 1.0 \text{ m}^2/\text{day}$, $\rho_s = 1.025 \text{ M/L}^3$, $\rho_f = 1.0 \text{ M/L}^3$, mean of $Y=\mu_Y = 2.0$, standard deviation of $Y=\sigma_Y = 2.5$, $R=0.0$ (no rainfall), and Y is the logarithm of the hydraulic conductivity. The well coordinates and their upper and lower

limits of pumping rates are shown in table 2. The upper limits of pumping is restricted with equipment and operational conditions. The lower limits are determined from economic operation of the well.

The following data were used with genetic algorithm: number of individuals=200, maximum number of generations = 100, crossover ratio=0.75, mutation ratio=0.15, seed number = -90, penalty= $2000\text{m}^3/\text{day}$ / intruded well.

Figure 4-a, gives an aerial view of the coast, locations of the pumping wells and magnitudes for logarithm of hydraulic conductivity (Y), that varied from -5.5 at white color zones to 7.6 at darkens zones. This indicate that as the square zones transfer to be darken its hydraulic conductivity increases. Figure 4-b, presents an aerial view for both the sharp interface toe and the well system. Shaded areas represent zones that have water table level (piezometric head) less than h_{loc} . These shaded zones can be classified in two groups:

Table 1, Rectangular coordinates, calculated pumping rates and corresponding analytical ones for well system.

INPUT DATA			OUTPUT RESULTS	
Well Number	X-Coordinate (m)	Y-Coordinate (m)	Calculated Optimal Discharge m^3/day	Analytical Optimal Discharge m^3/day
1	4000.0	250.0	452.26	461.5
2	4000.0	750.0	468.53	461.5
3	4000.0	1250.0	449.007	461.5
4	4000.0	1750.0	470.0	461.5
5	4000.0	2250.0	444.786	461.5
6	4000.0	2750.0	462.1609	461.5
7	4000.0	3250.0	463.202	461.5
8	4000.0	3750.0	450.659	461.5
9	4000.0	4250.0	460.471	461.5
10	4000.0	4750.0	459.08	461.5
Optimal Total Pumping Rates m^3/day (Q_{optimal})			4580.1559	4615

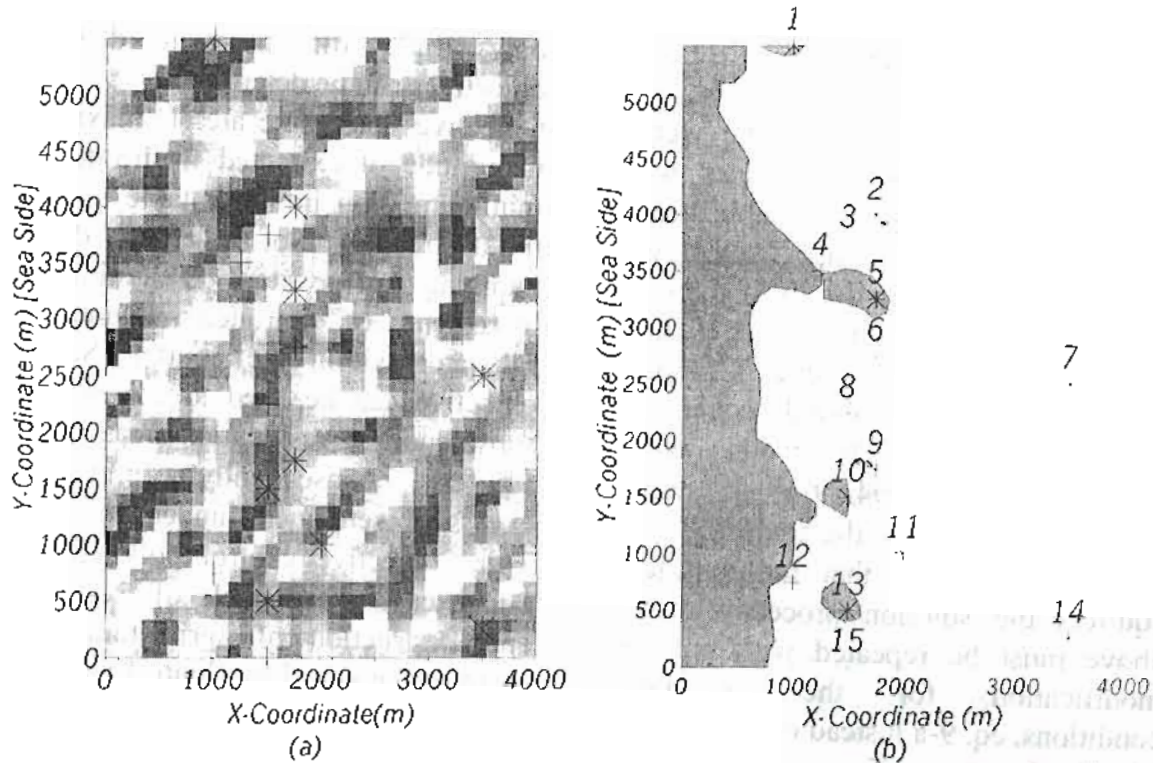


Figure 4. Layout of the studied aquifer, a) logarithm of hydraulic conductivity and well system, b) salt water zone and well system (+ switch off well, * working well).

Table 2. Pumping well, input data and optimal solutions (case study I)

INPUT DATA					OUTPUT RESULTS	
Well No.	X-coordinate (m)	Y-coordinate (m)	Lower Limit of Pumping (m ³ /day)	Upper Limit of Pumping (m ³ /day)	Optimal Solution Without Rainfall (m ³ /day)	Optimal Solution With Rainfall (m ³ /day)
1	1000	5500	150	600	600	600.0
2	1700	4100	150	1300	1162.129	1300.0
3	1500	3850	150	1100	0.0	0.0
4	1200	3400	150	800	0.0	0.0
5	1700	3200	150	1300	296.9	306.26
6	1800	2700	150	1400	0.0	0.0
7	3500	2500	150	1500	1357.5	1500.0
8	1600	2200	150	1200	0.0	177.309
9	1600	1800	150	1200	176.6	151.6
10	1500	1400	150	1100	150.0	161.0
11	2000	1000	150	1500	264.0	224.87
12	1000	800	150	600	0.0	0.0
13	1600	500	150	1200	604.0	522.67
14	3600	200	150	1500	590.0	1136.92
15	1400	0	150	1000	0.0	0.0
Optimal Total Pumping Rates m ³ /day (Q _{optimal})					5201.79	6080.631

Figure 6, shows the trade off contours between the three objective functions. This figure is used in order to analyze the interaction between the objectives and is very useful in the decision making process. It may enhance the selection decision maker's ability to select the best compromise solution. Depending on economic, environmental, social, and political aspects the decision maker can choose a favorable solution. For example, if maximum allowable drop in water table level is taken equal to $4.0m$ and the maximum allowable ratio for increment of intruded volume of saltwater is assumed equal to 2.0 , the optimal solution for the extracted freshwater will be equal to $4600m^3/day$, see figure 6.

It is observed from fig. 6, that maximum extracted water increases rapidly as maximum permissible drop in water table level increases or with

increasing the permitted volume of intruded seawater within the aquifer.

If the present problem is subjected to only two objective functions, e.g., maximum extraction of freshwater and minimum drop of water table level, the trade off curve between these two functions can be determined directly from fig. 6. This can be achieved by taken the constrain of the third objective function at its upper limit (at increment ratio equal to 3.75).

Figure 7-a shows the trade off curve between only two objectives functions Z_1 (maximum magnitudes of extracted freshwater(m^3/day)) and Z_2 (maximum drop in water level (m)), figure 7-b also shows trade off curve between objectives functions Z_1 and Z_3 (maximum increment ratio in intruded volume of sea water). It can be noticed that objective function Z_2 has more influence than function Z_3 .

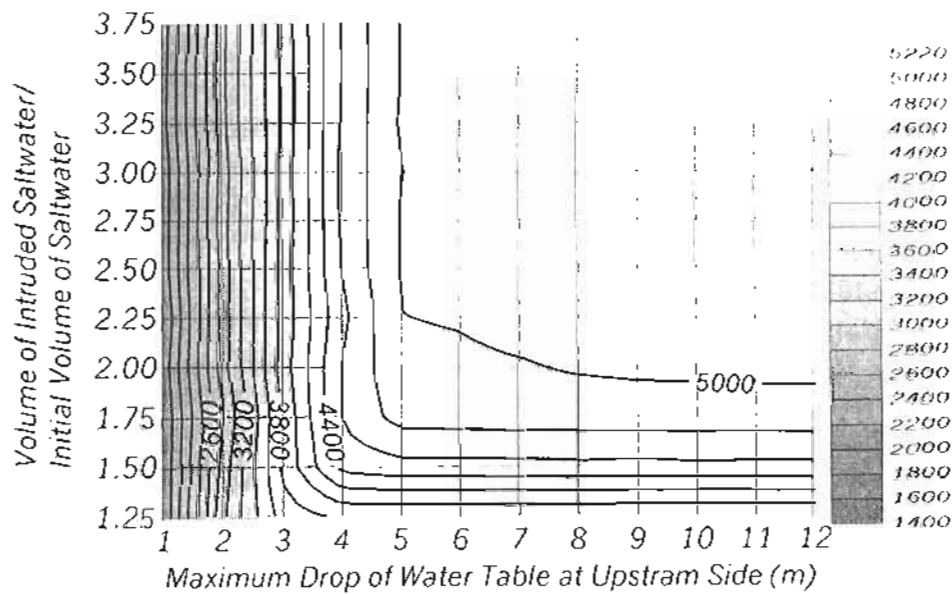


Figure 6 Trade off contours between maximum magnitudes of extracted freshwater(m^3/day) against maximum drop in water level(m) and maximum increment ratio of intruded volume of sea water.

9- Summary and Conclusions

The FEM was used to simulate the hydraulic response of steady heterogeneous coastal aquifer subjected to pumping from well system by taking into consideration Strack's formulation.

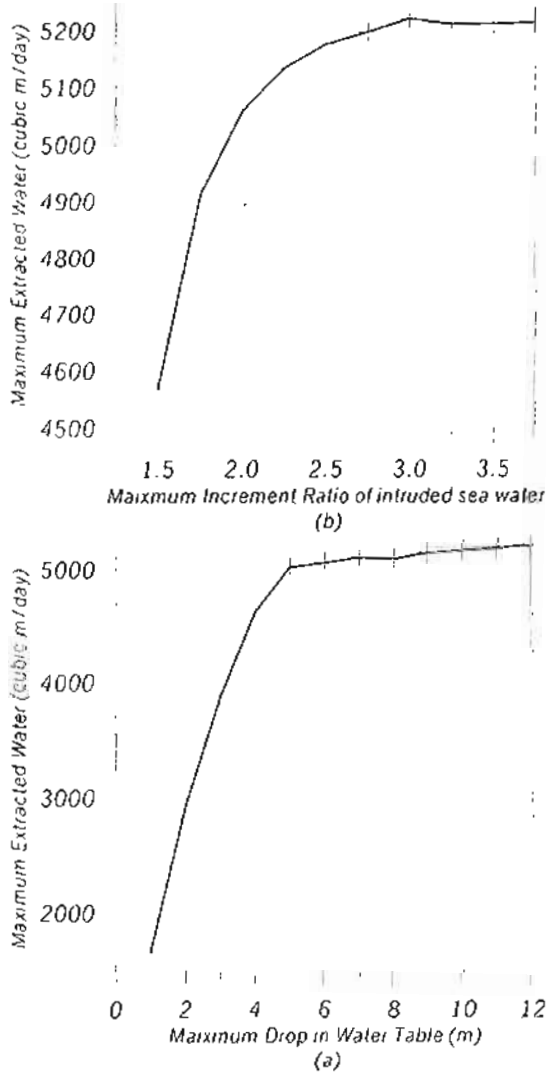


Figure 7. Trade off curves between a) maximum magnitudes of extracted freshwater(m^3/day) against maximum drop in water level(m), and b) maximum magnitudes of extracted freshwater(m^3/day) against maximum increment ratio in intruded volume of sea water.

This eliminate the non-linearity of the fundamental equations that considers the piezometric head as a dependent variable. GA method was applied to solve the optimization problem of

single-objective function (maximum permissible extraction of freshwater from the well system). To decrease the computation time the conductance/stiffness matrix was inverted once and used repeatedly through the optimization process. A Fortran program was written to apply the present model. The model verified against analytical problem with homogenous properties, then it used to solve heterogeneous coastal aquifer under single-objective function for two situations, without and with rainfall. The same aquifer was restudied under three objective functions. The constraints technique was applied to simplify that problem to a single-objective problem. The simplified problem was solved several times under varied constraints to find the trade off contours between different objective functions. The generated trade off contours enhance the decision maker's ability to select the best development policy from a set of alternative polices by considering technological, financial, and legal constrains.

Optimization problems handled in this work are nonlinear, so it is not suitable to solve these problems using the gradient methods, that usually stop at local minimums. GA method used to deal with the present problems. This stochastic nonlinear optimization method can easily overcome both of non-linearity and discontinuity through the optimization space.

Using the LU-decomposition method to inverse the Conductance matrix made the solution of the multi-objective problem possible, without matrix inversion that problem needs several days for the computation process.

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